Simulation Study for Effects of Tick Sizes Difference in Stock Markets Competition

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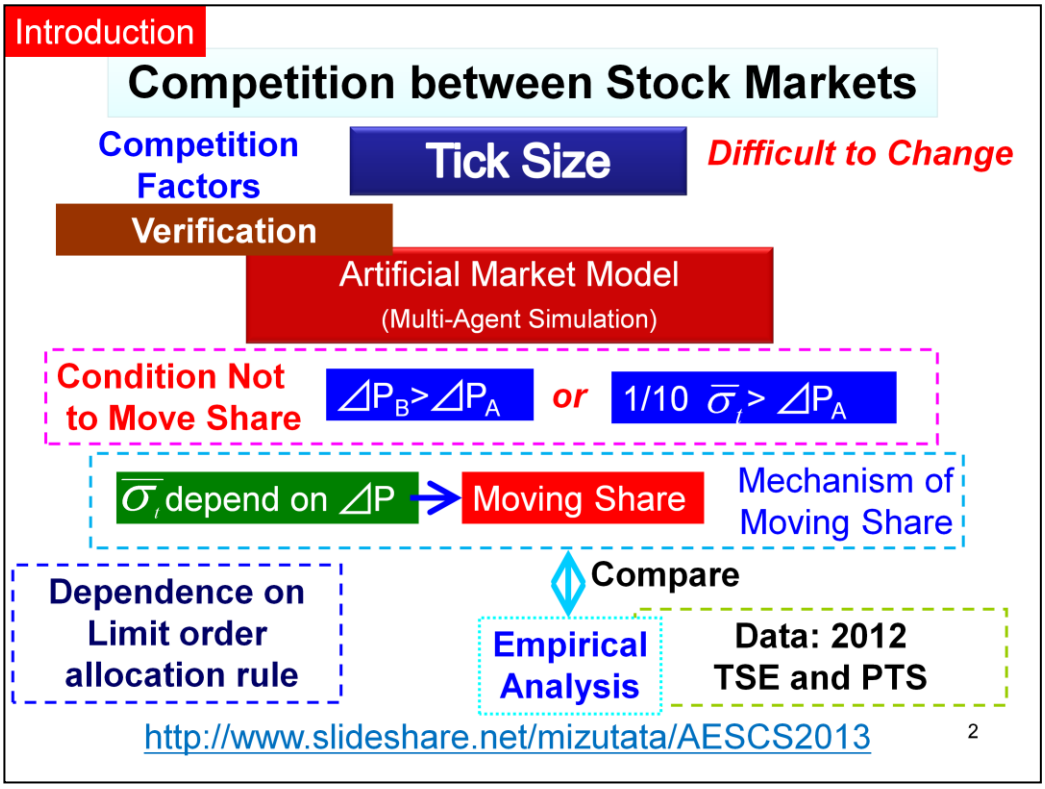
Thank you very much. I'm Takanobu MIZUTA from SPARX Asset Management in Tokyo.

I'm also belonging to The University of Tokyo.

Today, I'm going to give a presentation under the title of This.



Introduction.

Recently, many low-cost Stock Market are born, and the Competition between Stock Markets is heated up.

There are many Competition Factors. It is said that “Tick Size” is very important Factor. (次のページ)

(戻ってきて)It is difficult to change Tick Size, therefore, It is very hard that we discuss about effect of Tick Size only using experiments.

In this study, we built Artificial Market Model, Multi Agent Simulation, to investigate The effect of Tick Size in the Competition.

At First, I will mention about our model and verify our model.

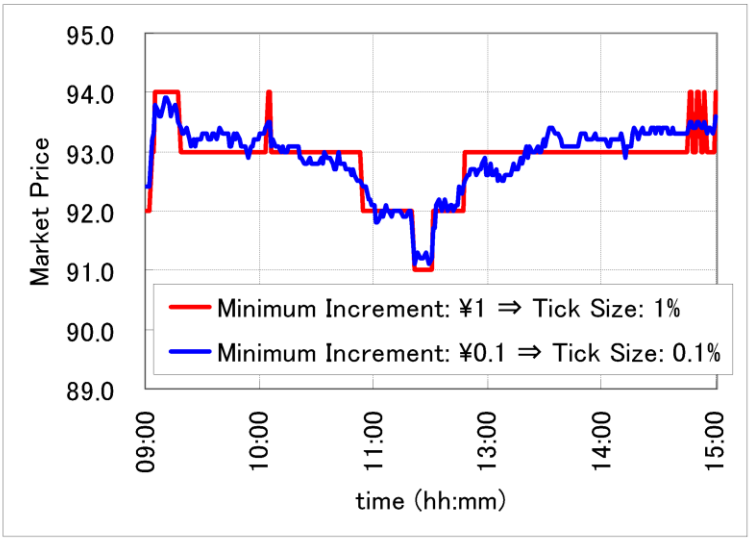
Next, I will show the conditions of Tick Sizes not to Moving Share of Trading Volume, and describe Mechanism of Moving Share,

Relation ship volatility and tick size.

Finally I will compare the simulation results to Empirical Analysis.

What is Tick Size?

Here, we define Tick Size ⊿P = Minimum Increment / Price



Difference of 1% Return is Serious Problem for some Investors
⇒ They prefer Stock Market has Smaller Tick Size ⊿P

Tick Size is minimum unit of price change, minimum increment per price.

This figure shows time evolution of market prices, 2 cases of different tick sizes.

Red line, minimum increment is one yen, prices are about 100 therefore, tick size is one %.

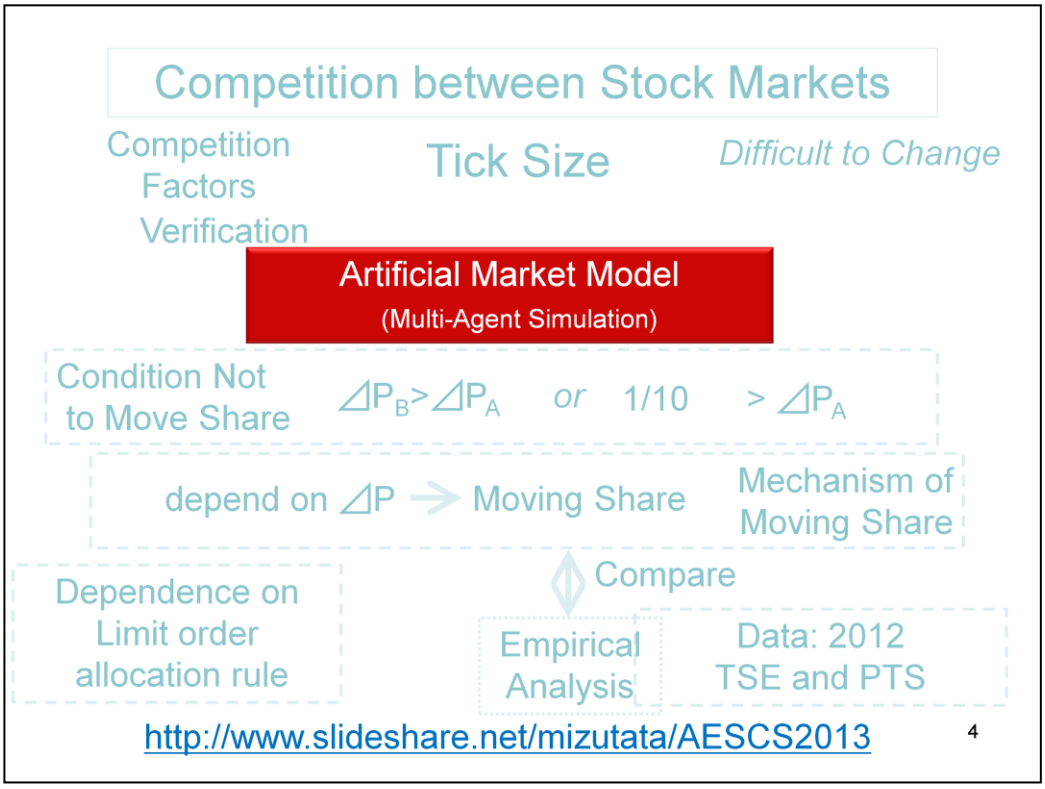
Prices are very roughly moving.

Blue line, minimum increment is 0.1 yen, tick size is 0.1%.

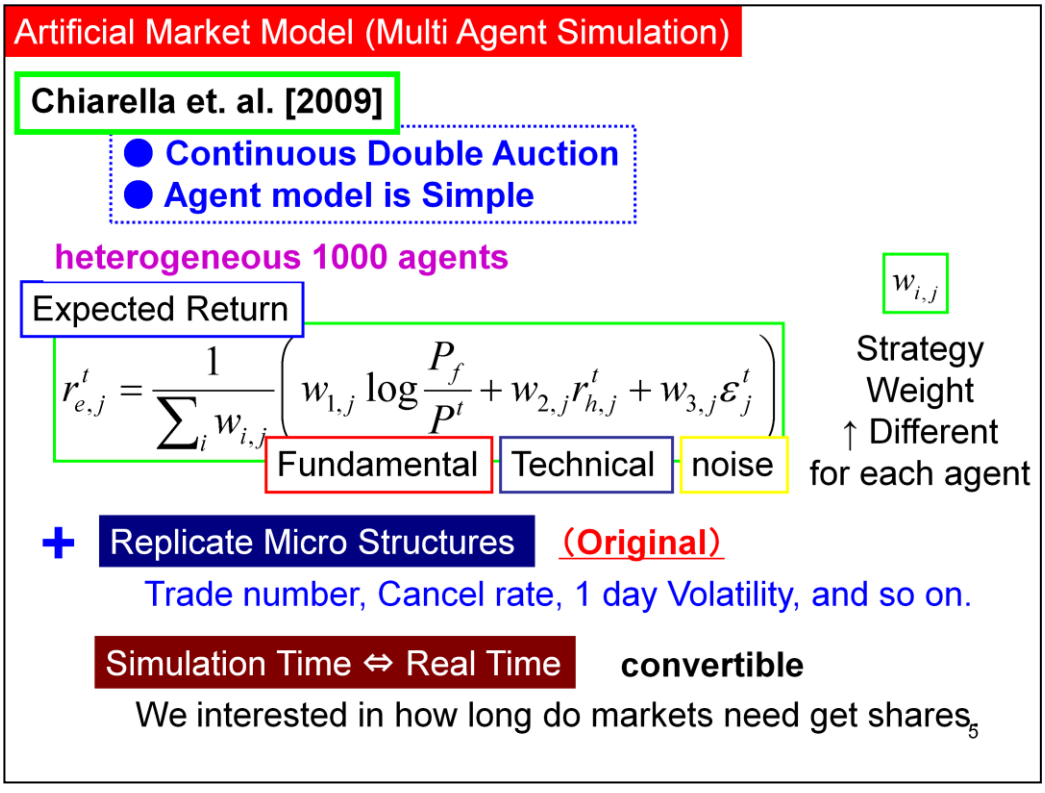
Prices are very smoothly moving.

Difference of 1% Return is Serious Problem for some Investors

They prefer Stock Market has Smaller Tick Size



First, I will describe our artificial market model.



We built an artificial market model on basis of Chiarella et. al. 2009.

Pricing mechanism is Continuous Double Auction. We need to implement market selection model.

Agent Model is Simple. This is to avoid arbitrary result.

We think Artificial Market Models should explain Stylized Facts as Simply as possible.

There are heterogeneous 1000 agents. All agents calculate Expected Return using this equation,

And, the strategy weights are different for each agent (次のページ)

(戻ってきて)Third term is noise.

We also replicate micro structures, this is our Original.

We also fit our model, not only traditional satirized facts but also micro structures' statistic,

trade number, cancel rate, 1 day return 1 day Volatility, and so on.

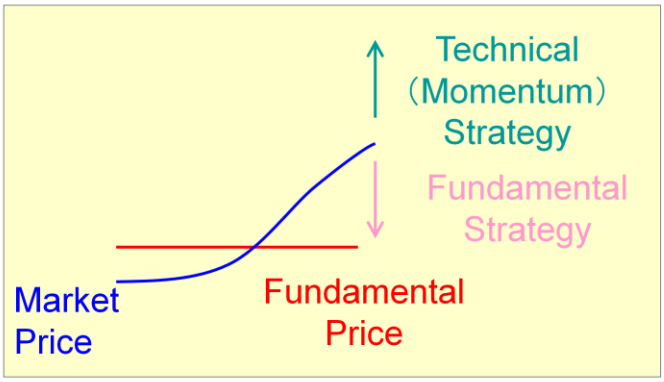
Volatility means standard deviation of price return.

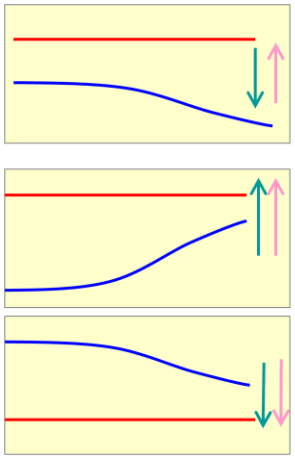
From this fitting, we can convert time in simulation, tick time and real world time.

This convertible is important because we interested in how long do markets need get shares of trading volume.

Terms of Fundamental Strategy, Technical Strategy

* **Fundamental Strategy Term**
  Fundamental Price ＞ Market Price ⇒ expects + return
  Fundamental Price ＜ Market Price ⇒ expects - return
* **Technical Strategy Term**
  Historical Return ＞ 0 ⇒ expects + return
  Historical Return ＜ 0 ⇒ expects - return





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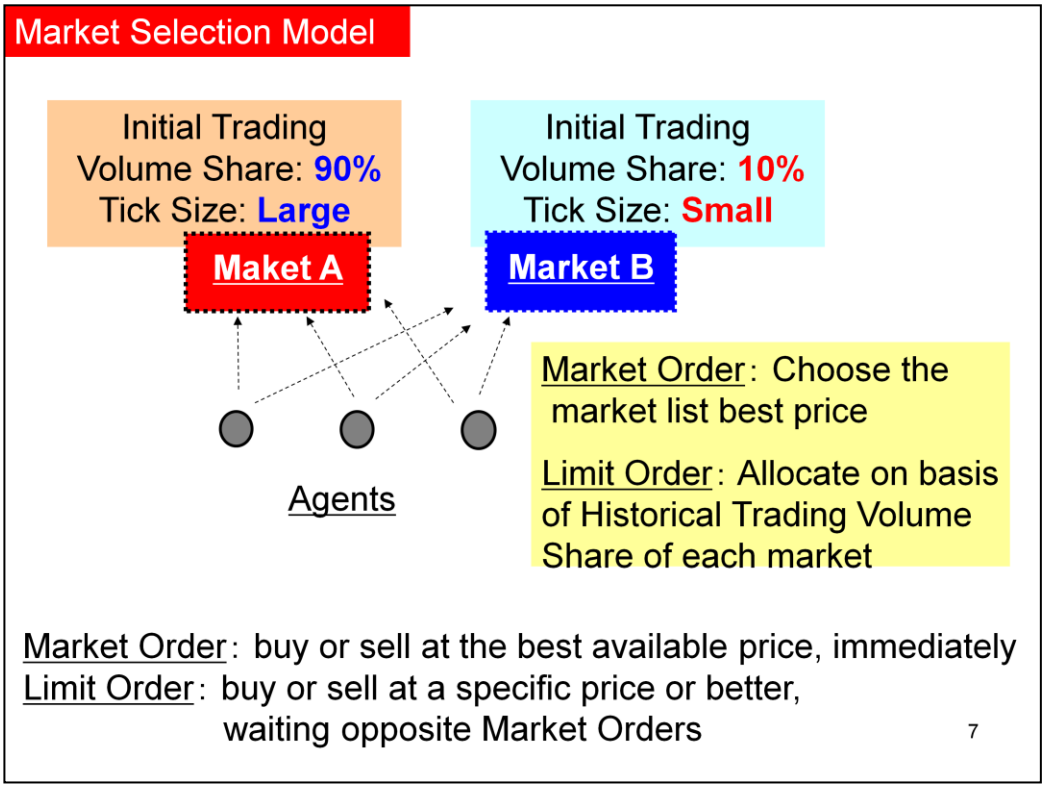
First term is a Fundamental Strategy: When the market price is smaller than the fundamental price, an agent expects a positive return , and vice verse.

Second term is a technical strategy: When historical return is positive, an agent expects a positive return, and vice verse.



Next, I describe market selection model.

Market A have 90% initial trading volume share and Large tick size.

Market B have 10% initial share and Small tick size.

When agents order Market order, they choose the market list best price.

When agents order Limit Order, they allocate orders on basis of Historical Trading Volume share of each market

Market Order means buy or sell at the best available price, immediately

Limit Order means buy or sell at a specific price or better, waiting opposite Market Orders

Market Selection Model (example)

Order Book

Market A		
Sell	Price	Buy
84	101	
176	100	
	99	204
	98	77

Market B		
Sell	Price	Buy
1	99.2	
2	99.1	
	99.0	3
	98.8	1

Limit Orders

(1) Buy ¥98: Allocate on basis of Historical Trading Volume Share of each market

(2) Buy ¥99.1: Market B
↑can buy ¥99.1 at Market B, immediately

(3) Buy ¥100: Market B
↑can buy ¥99.1 at Market B, best price

Market B will take Trading Volume share because of (2), (3)

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I show example, Now, order books are like this,

Case (1), agent buys at ¥98, this order become Limit Order, therefore,

Allocate on basis of Historical Trading Volume Share of each market

Case (2), agent buys at ¥99.1, he can buy ¥99.1 at Market B immediately, however, he can not at Market A.

Therefore, He choose Market B.

Case (3), agent buys at ¥100, both market A and B, he can buy immediately, Market A, ¥100, Market B, ¥99.1.

Market B list better price, therefore, he choose Market B

Thorough these process, Market B will take trading volume share because of case (2) and case (3)

Allocate on basis of Historical Trading Volume Share

Wa : Probability an agent choose Market A

Ta , Tb : Trading Volume of Market A or B within last t_{AB}

$$Wa = \frac{Ta}{Ta + Tb}$$

(1) t_{AB}=5 days

(2) Various t_{AB} → **Dependence on Limit order allocation rule**

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We describe detail of Allocate on basis of Historical Trading Volume Share

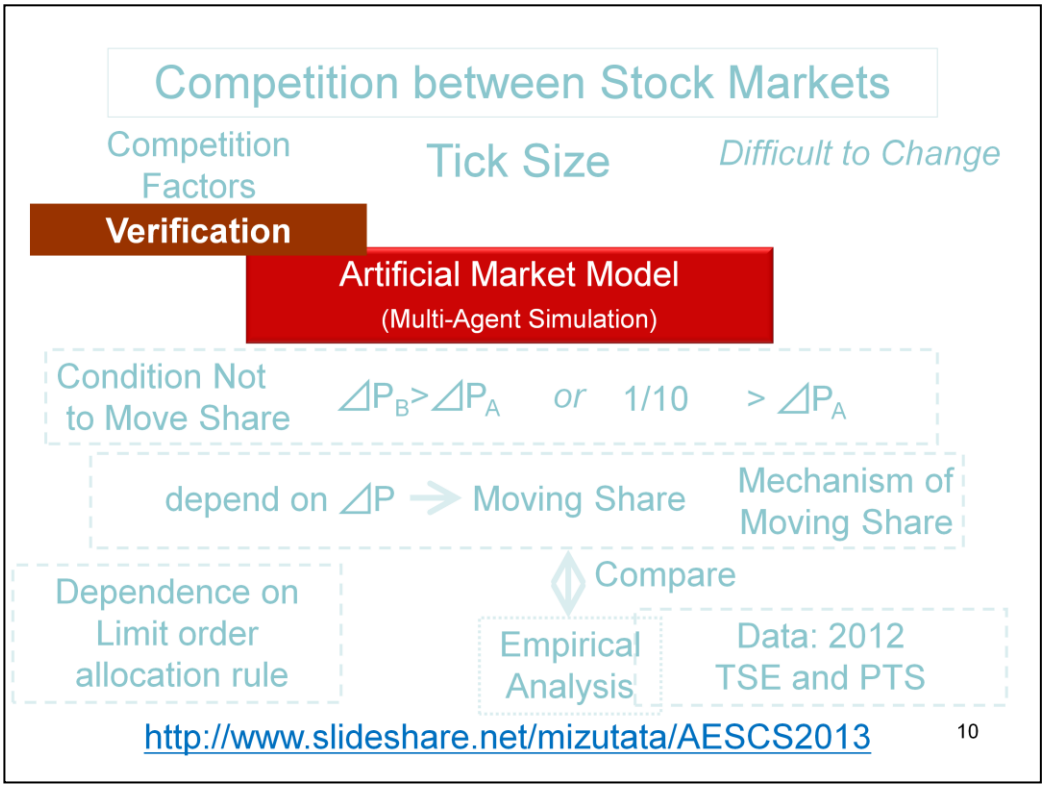
Wa is Probability an agent choose Market A

Ta and Tb are Trading Volume of Market A or B within last tAB

Wa is defied like this, weight of Historical trading volume.

Tab is important parameter. At first, we investigate simulation runs fixing tab=5 days.

After that, we change tab variously, to investigate dependence on tab, limit order allocation rule.



Here, I will show about verification of our Model

Stylized Facts

	tick size(%)	0.0001%	0.001%	0.01%	0.1%	1%
about trading	trade rate	23.5%	23.5%	23.4%	23.1%	22.1%
	cancel rate	26.2%	26.2%	26.3%	26.6%	27.6%
	number of trades / 1 day	6,361	6,358	6,345	6,279	6,081
standard deviations	for 1 tick	0.05%	0.05%	0.05%	0.06%	0.16%
	for 1 day (20000 ticks)	0.59%	0.56%	0.57%	0.57%	1.15%
kurtosis		1.50	1.48	1.45	1.10	1.81
autocorrelation coefficient for square return	lag 1	0.229	0.228	0.228	0.210	0.025
	2	0.141	0.141	0.141	0.120	0.013
	3	0.109	0.108	0.108	0.090	0.008
	4	0.091	0.091	0.091	0.075	0.006
	5	0.078	0.078	0.078	0.064	0.004

Replicate Fat-Tail and Volatility-Clustering

$\overline{\sigma}_t = 0.05\%$

Volatility at tick size small

\+ Replicate Micro Structures (Original)

Trade rate, Cancel rate, 1 tick and 1 day volatility

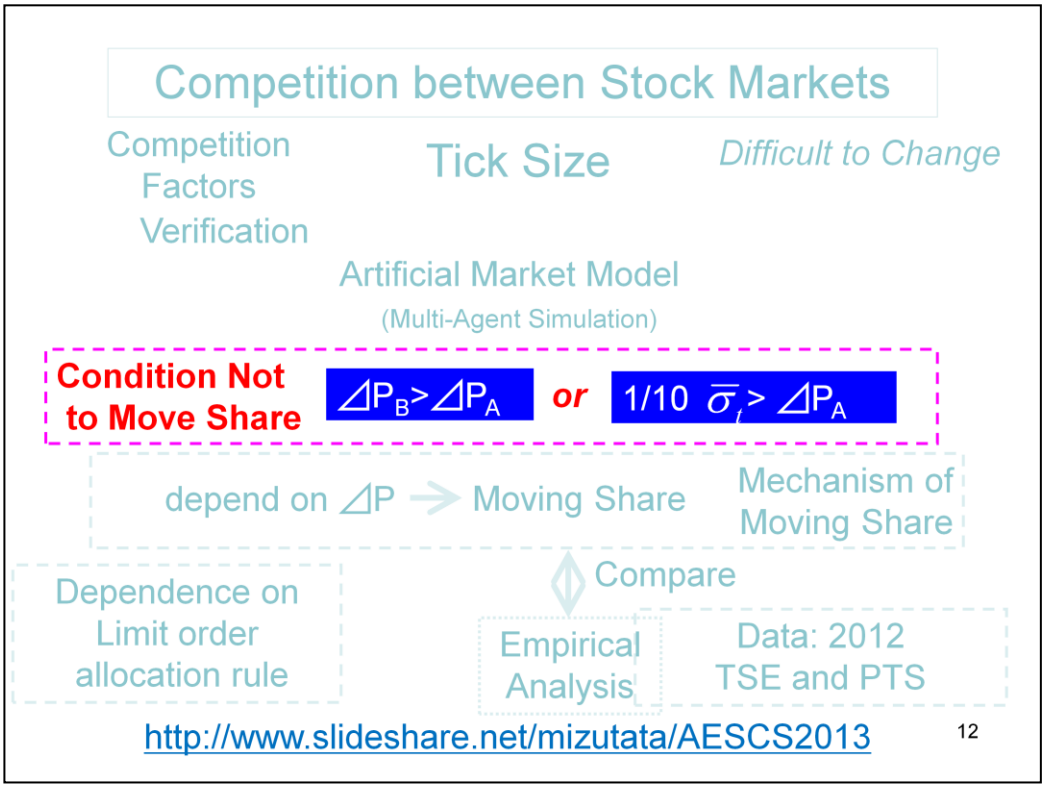
Simulation Time ⇔ Real Time **convertible**

We interested in how long do markets need get shares.

This table list traditional Stylized Facts and statistics of Micro Structures for various Tick Size, in the case that there is One Market.

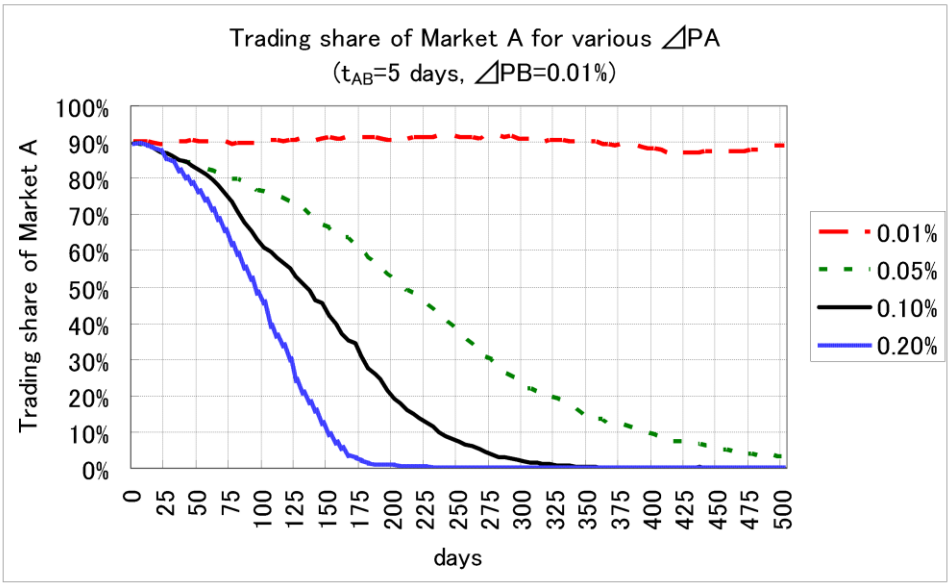
In all cases, both kurtosis and autocorrelation for square returns for all lag are positive.

This means that all cases replicate Traditional stylized facts: fat-tail and volatility-clustering.
Trade rate, Cancel rate, 1 tick and 1 day volatility are very similar to those of real stock markets.
Therefore the model replicate micro structures.
We found that Tick Time 20,000(twenty thousand) in simulation correspond to 1 day in real world.
Therefore we can convert simulation time and real time.
We emphasis that 1 tick volatility at Tick Size is enough small, 0.05% is very important number.
Whether Tick Size is bigger or smaller than 1 tick volatility, effects aspect of moving share of trading volume.
I define as Sigma t Var.



Next I describe the condition Not to Move Share of Trading Volume

Tick Size of Market B ⊿PB=0.01%, Tick Size is not small



Tick Size of Market A, ⊿PA is larger,
Market A is taken trading volume share faster

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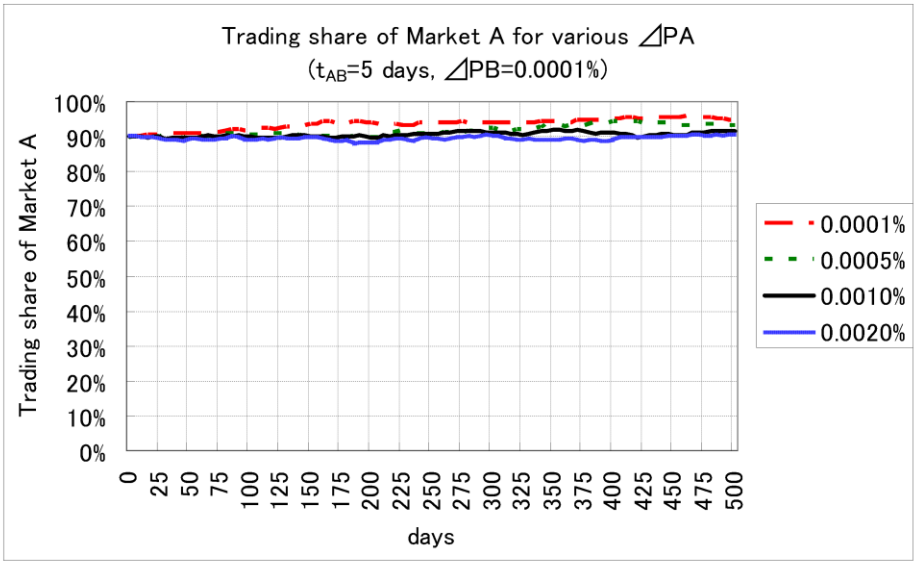
This figure shows time evolution of shares of trading volume of Market A for Various Tick Size ⊿PA

Tick Size of Market A is larger, Market A is taken trading volume share from Market B, faster.

⊿PB=0.0001%, Tick Size is enough small



Market B can hardly take the share in spite that ⊿PA is very larger than ⊿PB

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This figure also shows time evolution of shares in the case Tick Size is enough Small.

In this case Market B can hardly take the shares in spite that Tick Size of Market A is very larger. 20 times, than that of Market B.

Tick Size Condition Not to Move Share

share of Market A at 500 days		Market B Tick Size ⊿PB										
		0.0001%	0.0002%	0.0005%	0.001%	0.002%	0.005%	0.01%	0.02%	0.05%	0.1%	0.2%
Market A Tick Size ⊿PA	0.0001%	90%	90%	91%	91%	92%	94%	97%	99%	100%	100%	100%
	0.0002%	90%	90%	90%	91%	91%	94%	97%	99%	100%	100%	100%
	0.0005%	89%	90%	91%	91%	92%	94%	96%	99%	100%	100%	100%
	0.001%	89%	89%	90%	90%	92%	94%	97%	99%	100%	100%	100%
	0.002%	87%	88%	89%	89%	91%	93%	97%	99%	100%	100%	100%
	0.005%	84%	85%	85%	84%	87%	92%	96%	99%	100%	100%	100%
	0.01%	75%	76%	76%	77%	78%	83%	92%	98%	100%	100%	100%
	0.02%	53%	52%	53%	54%	54%	59%	70%	93%	100%	100%	100%
	0.05%	5%	5%	4%	5%	5%	5%	6%	23%	93%	100%	100%
	0.1%	0%	0%	0%	0%	0%	0%	0%	0%	0%	94%	100%
	0.2%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	96%

Condition Not to Move Share $\Delta P_B > \Delta P_A$ *or* $1/10\ \bar{\sigma}_t > \Delta P_A$

Condition to Rapidly Move Share $\bar{\sigma}_t < \Delta P_A$ $\bar{\sigma}_t = 0.05\%$

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This table lists trading volume shares of Market A at 500 days various Tick Size of Market A and B.

In the area Tick Sizes of Market A are smaller than those of Market B, upper right dashed line, Market B did not take the shares.

And in the area Tick Size of Market A is smaller than one-tenth of Sigma t Bar, which is 0.05% as I mentioned,

Upper heavy solid line, Market B did not take the shares, even if Tick Size of Market A is bigger than that of Market B.

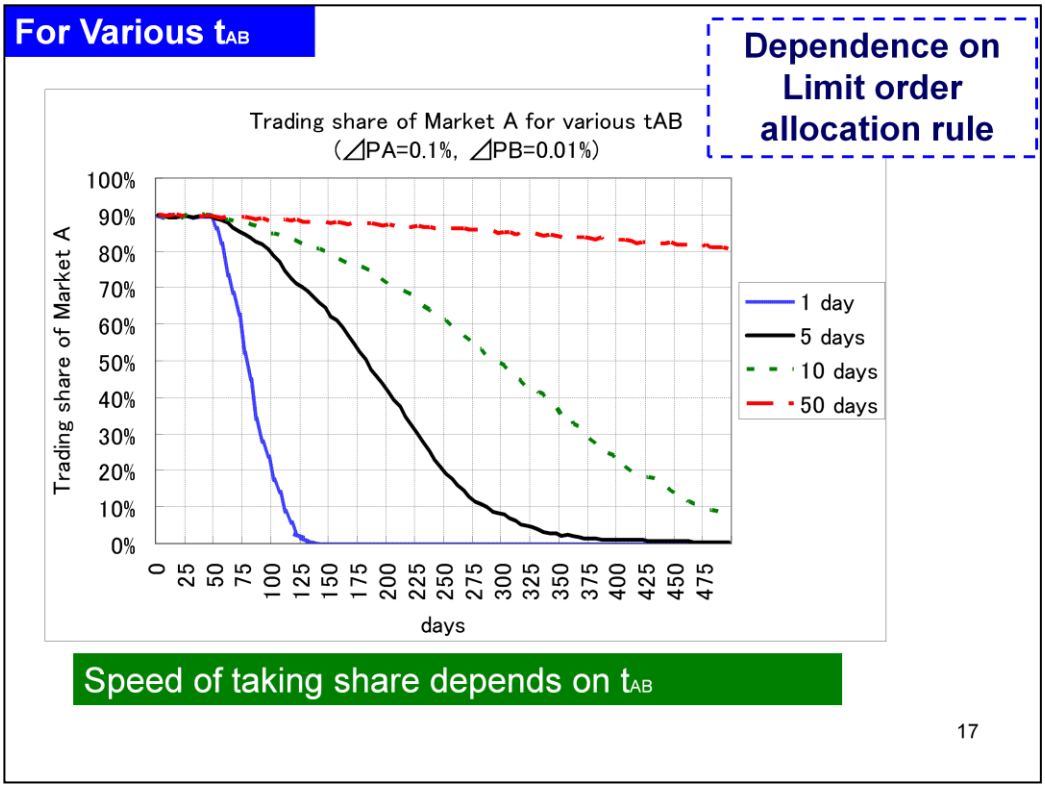
These are the Condition Not to Move Share to Market B.

I emphasis that competitions under too small tick sizes do not effect taking trading volume shares.

On the other hand, Tick Size of Market A is bigger than Sigma t bar, under double solid line, Market B took Share very Rapidly.

Therefore, when tick sizes are bigger than Sigma t Bar, Tick Sizes Competitions are very important.

Next I talk about Dependence on Limit order allocation rule



This figure shows time evolution of shares of the market A for various tAB where ΔPA = 0.1%, ΔPB = 0.01%.

We found that tAB is smaller, market B takes shares faster.

It means that Speed of taking share depends on tAB.

Tick Size Condition Not to Move Share

share of Market A at 500 days		Reference days t_{AB} 1	2	5	10	20	50
Market A Tick Size ⊿PA	0.0001%	91%	89%	90%	90%	90%	90%
	0.0002%	89%	90%	90%	90%	90%	90%
	0.0005%	89%	88%	89%	90%	90%	90%
	0.001%	88%	88%	89%	90%	90%	90%
	0.002%	84%	86%	88%	89%	90%	90%
	0.005%	62%	73%	85%	87%	89%	89%
	0.01%	24%	53%	78%	85%	87%	89%
	0.02%	0%	12%	59%	77%	84%	88%
	0.05%	0%	0%	8%	44%	73%	85%
	0.1%	0%	0%	0%	10%	52%	80%
	0.2%	0%	0%	0%	0%	24%	74%

$\overline{\sigma}_t = 0.05\%$

Condition Not to Move Share

$\Delta P_B > \Delta P_A$ *or* $1/10\ \overline{\sigma}_t > \Delta P_A$

Condition to Rapidly Move Share

$\overline{\sigma}_t < \Delta P_A$ ⇒ **Not Rapidly**

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This table shows shares of market A at 500 days, for various Tick Size of Market A and tAB where Tick Size of Market B is enough small.

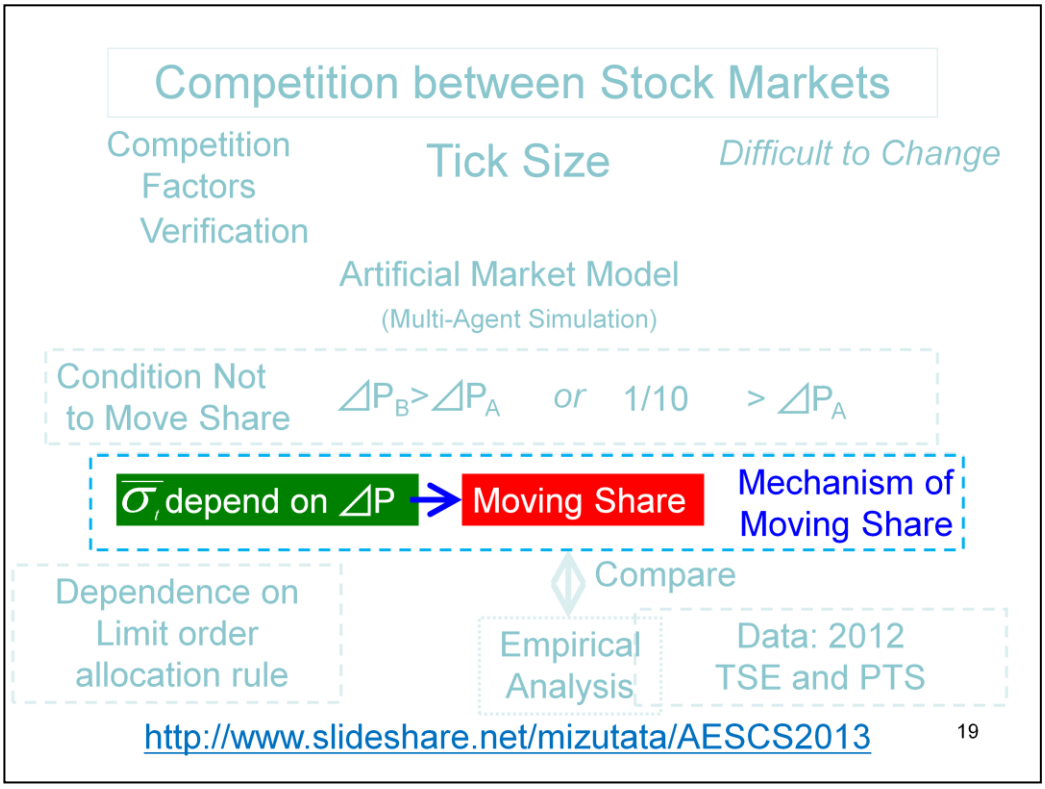
Tick Size of Market A is smaller than one of tenth Sigma t Bar, upper heavy line, for any tAB, Market B can not take shares.

Therefore, the Condition not to move share is not changed.

On the other hand, Tick Size of Market A is bigger than Sigma t Bar, under double solid line, in large tAB,

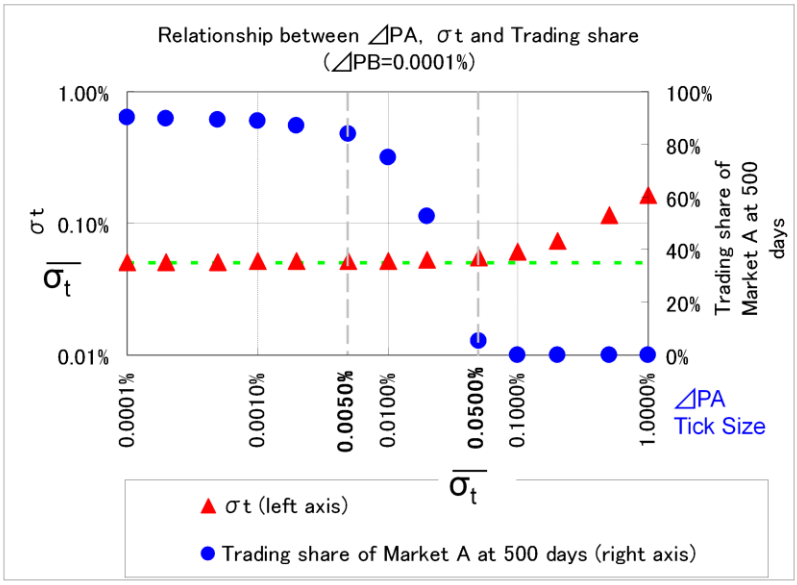
Market B take shares not Rapidly.

These Results shows that tab effects taking share Speed, but, do not change these qualitative conditions whether shares are move or not.



Next I talk about , Mechanism of Moving Share, relationship between Volatility and Tick Size

Relationship between σt and Share (⊿PB is enough small)



When σt depends on ⊿PA, Market A is taken share very Rapidly

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This figure shows relationship between 1-tick Volatility, Sigma t and Tick Size. Tick Size of Market B is enough small.

Red Delta is 1-tick volatility, σt Blue Circle is Share of Market A at 500 day.

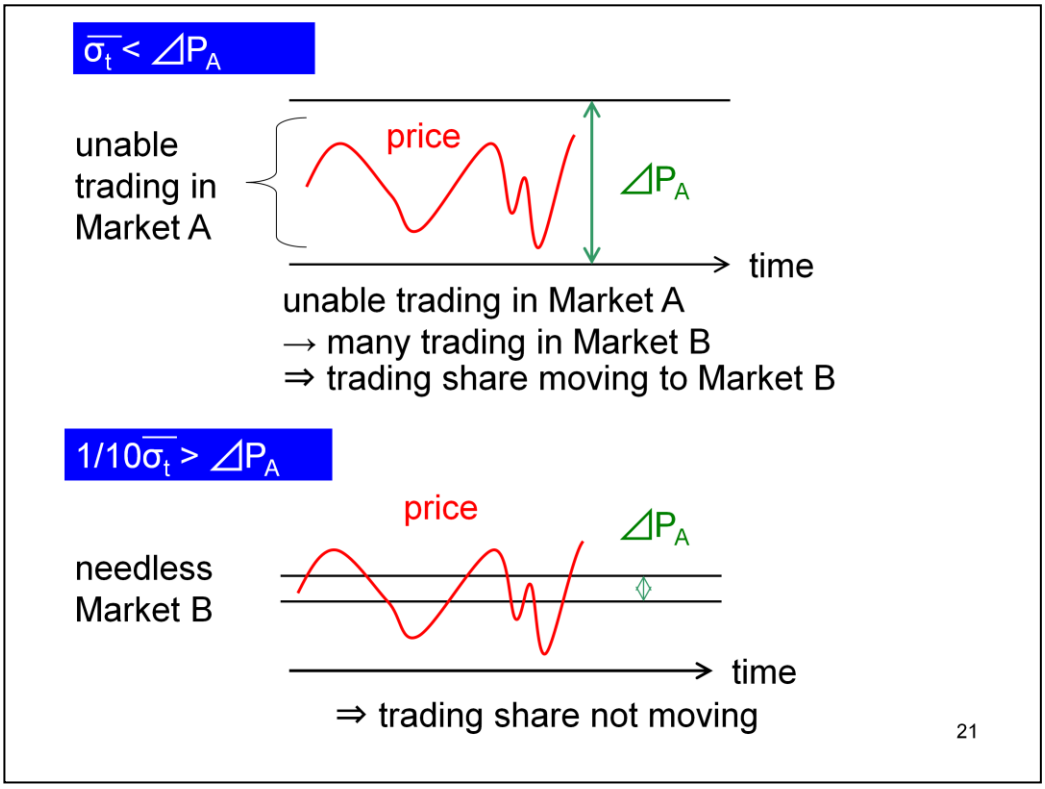
Horizontal axis is ⊿PA, Tick Size of Market A.

In Left Side from 0.05%, Tick Size is smaller than Sigma t Bar, Volatility similar equals to Sigma t Bar. Volatility does not depend on tick size.

On the other hand, in right side of Sigma t Bar, Tick Size is larger than Sigma t Bar, Tick Size is larger, Volatility is larger.

In this case, trading volume shares rapidly decease.

So, When σt depends on ⊿PA, Market A is taken share very Rapidly



Summary for a relationship between tick sizes and standard deviations of returns for 1 tick.

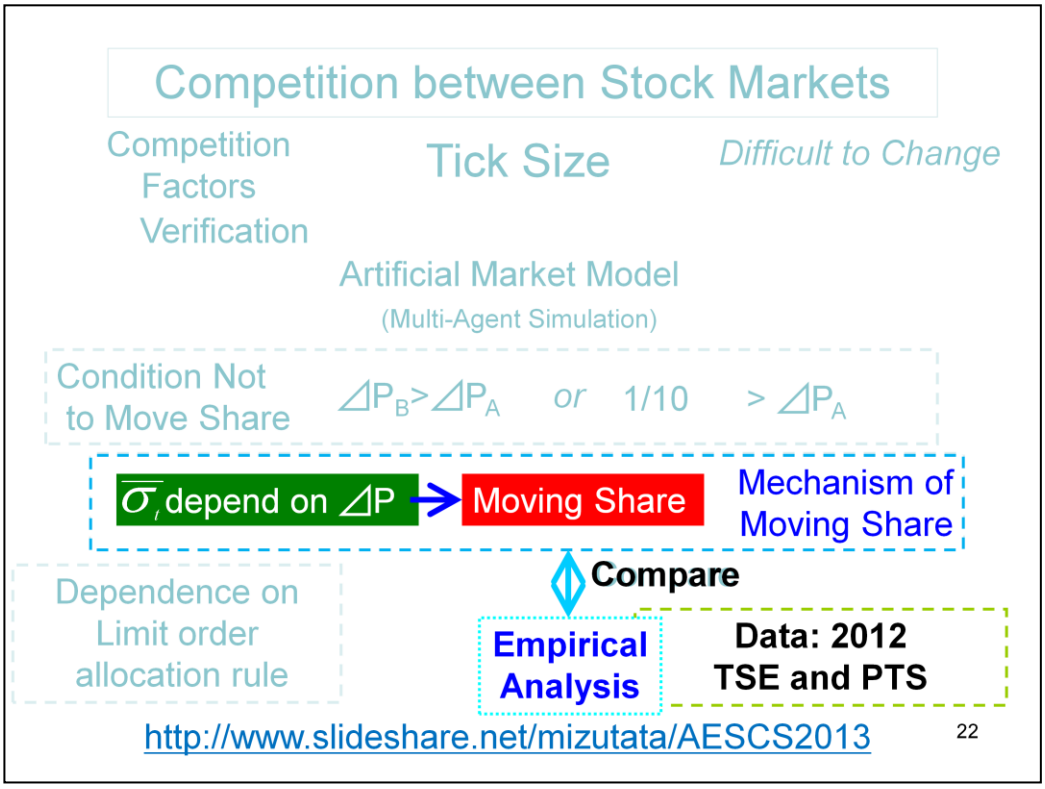
In the case Tick Size of Market A is larger than Volatility,

In these prices, agents can not trade in Market A.

Therefore, agents use Market B, and Market B will take Share of Trading Volume.

In the case Tick Size of Market A is enough smaller than Volatility,

Agents need not use Market B, therefore Market B will not take share.



Lastly, I Compare Empirical Analysis

Empirical Study

Data

Data Period: All business days in calendar year 2012

Universe: 439 stocks

Selected by TOPIX 500 index whole data period
they had same tick size for every month ends
they were traded every business days at least once

Horizontal Axis: Tick Size of TSE ⊿P for each stock

▲: standard deviation of 10 seconds return for each stock, σt

●: trading volume share in PTS for each stock

Summarize Markets:

Traditional Stock Exchanges:

Tokyo Stock Exchange, Osaka SE,
Nagoya, Fukuoka, Sapporo, and JASDAQ

PTS (Proprietary Trading System):

Japan Next PTS J-Market, Japan Next PTS X-Market,
and Chi-X Japan PTS

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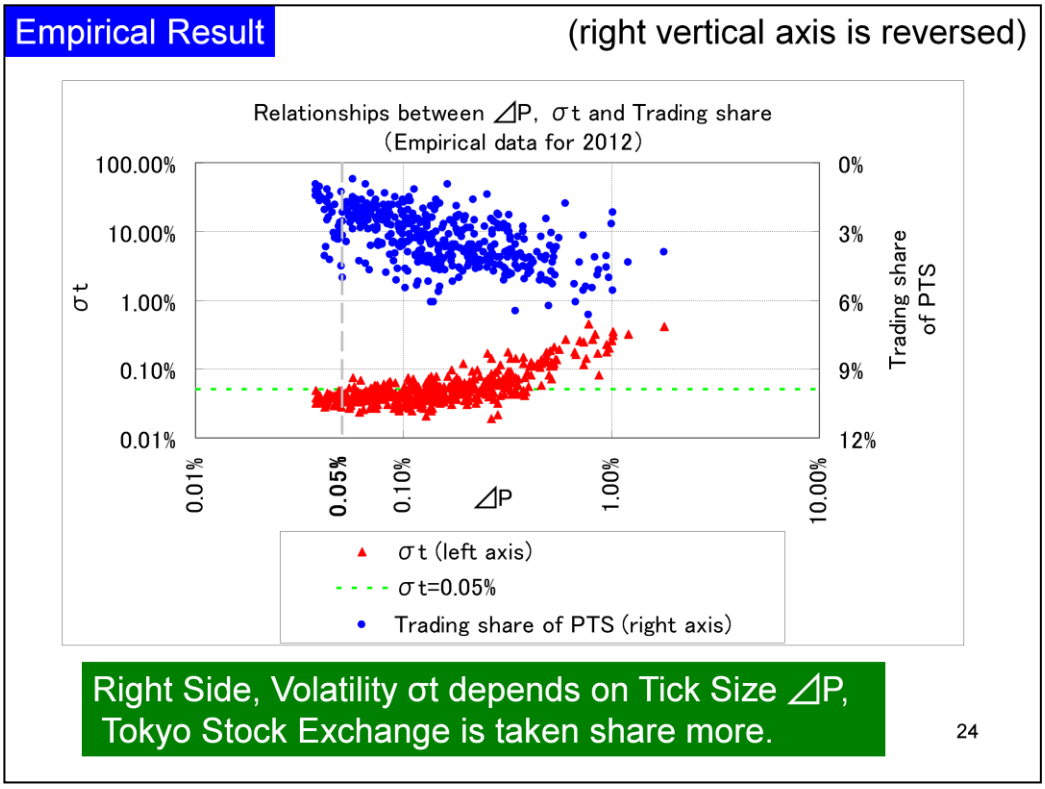


This figure shows result of Empirical Analysis.

Red Delta shows 10 Seconds Volatility, Blue Circle is Trading Volume Share of PTS, vertical axis is reversed.

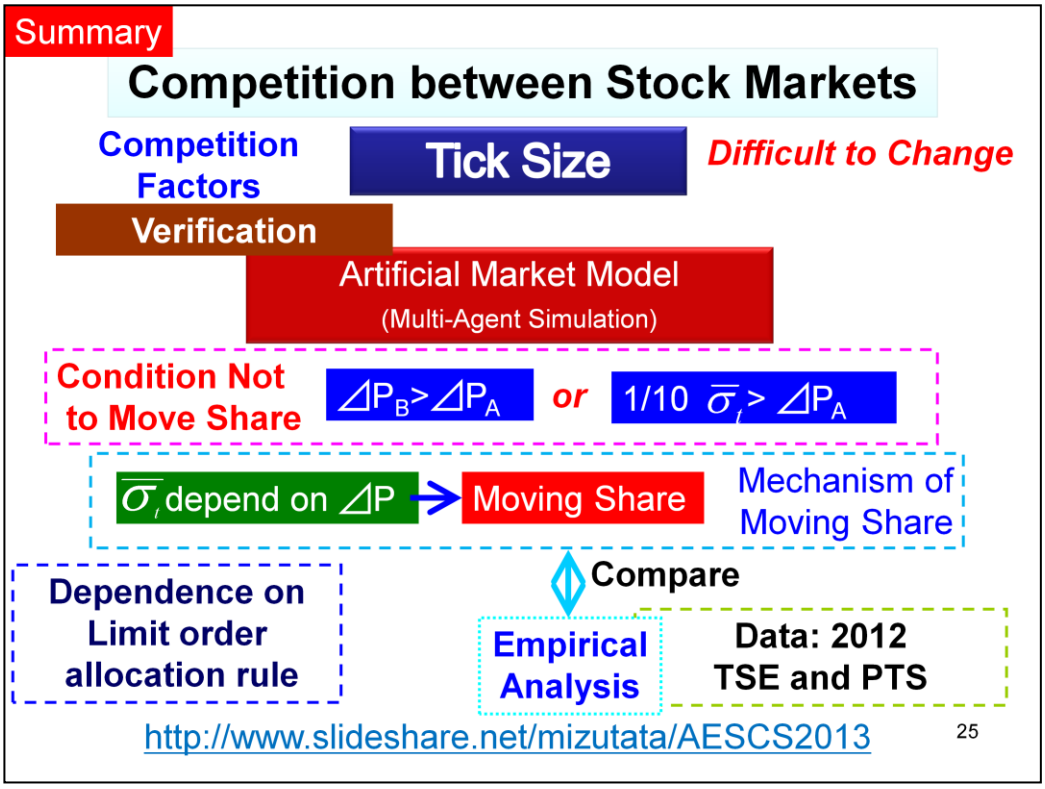
Horizontal axis is Tick Size of Tokyo Stock Exchange.

Note that Ticks Sizes of Tokyo Stock Exchange are 10 times Larger than those of PTS.

Therefore, Tokyo Stock Exchange is corresponding to Market A, and PTS is corresponding to Market B.

Right Side, Volatility σt depends on Tick Size ⊿P, Tokyo Stock Exchange is taken share more.

This Empirical Result is very similar tendency to Simulation Result as I showed.



I summarize this presentation

We investigate Tick Size Effect in the competition between stock markets using Artificial Market Model.

We found these condition Not to Moving Share.

When Tick Size is Larger than Volatility, Volatility depends on Tick Size, and the Market is taken share of trading volume by another Market.

We compared these simulation results with empirical data of Tokyo Stock Exchange.

We emphasize that these investigation will give us much implications for discussion

about adequate tick sizes markets should adopt.

Could you say that again? (もう一度、おっしゃっていただけますか？)

I don't quite understand your question. (ご質問の趣旨が良く分からないのですが)

Could you please rephrase your question? (ご質問を分かりやすく言い換えていただけますか)

So, you are asking me about.... (つまり、お尋ねの内容は...ですね)

I totally agree with you. (私も全くあなたと同意見です)

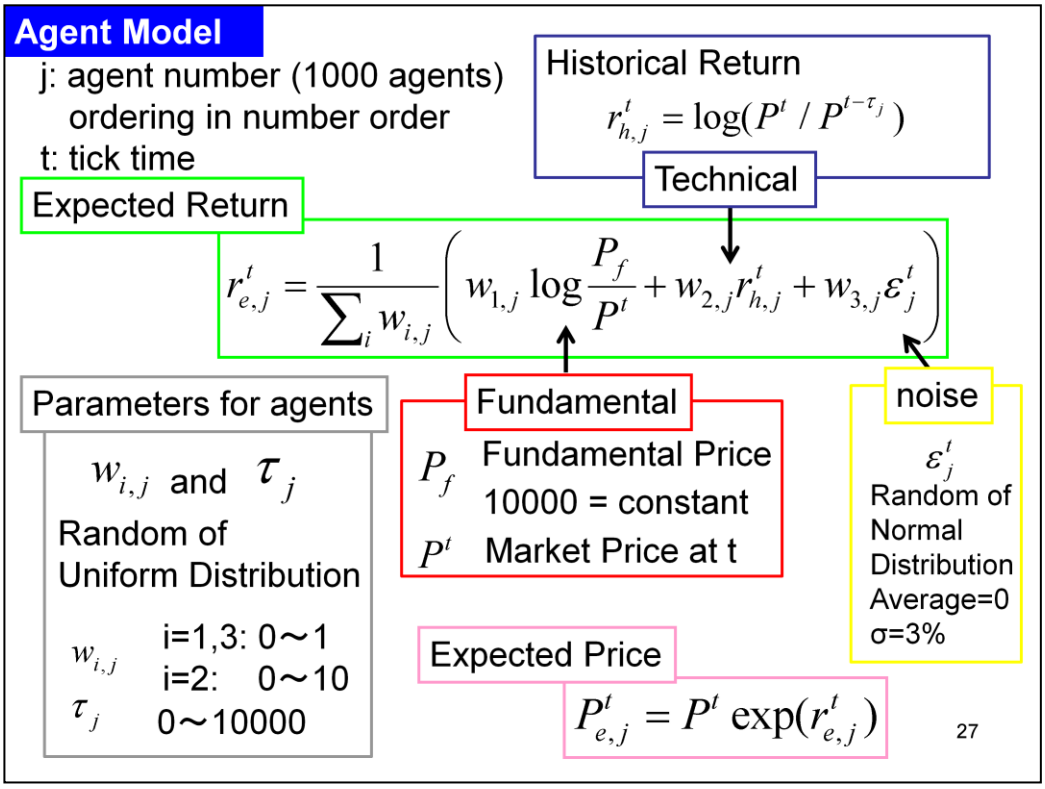
That's a very challenging question for me to answer. (それは私にとって非常に答えがいのある質問です)

That's a question I'm not sure I can answer right now. (そのご質問にすぐお答えできるかどうか分かりません)

It would require further research. (さらなる研究結果を待ちたい)

You are right on that point. (その点に関してはあなたが正しい)

Our method will not solve the problem. (我々の方法ではその問題は解決できない)



Next, I will describe agent model.

All agents calculate Expected Return using this equation.

First term is a Fundamental Strategy:

When the market price is smaller than the fundamental price, an agent expects a positive return , and vice verse.

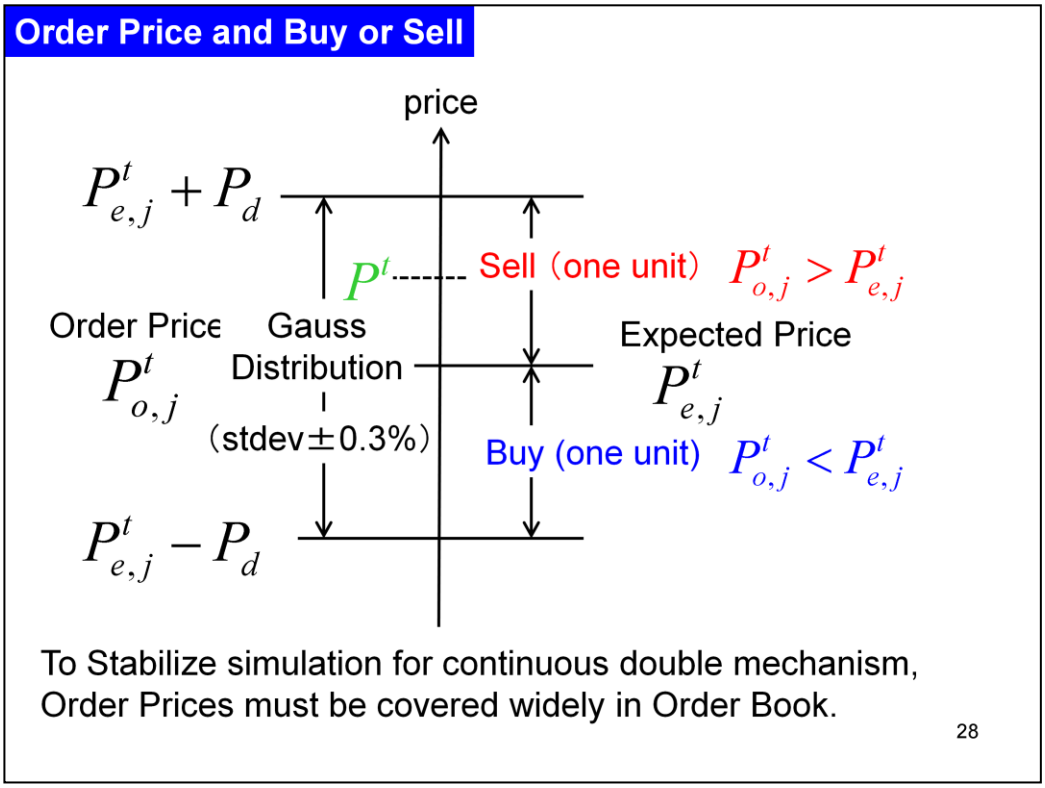
Second term is a technical strategy:

When historical return is positive, an agent expects a positive return, and vice verse.

Third term is noise,

After the expected return has been determined, an expected price is determined like this.

And, agents order base on this Expected Price.



Next, agents determine order price and, buy or sell.

To Stabilize simulation runs for the continuous double mechanism,

Order Prices must be covered widely in Order Book.

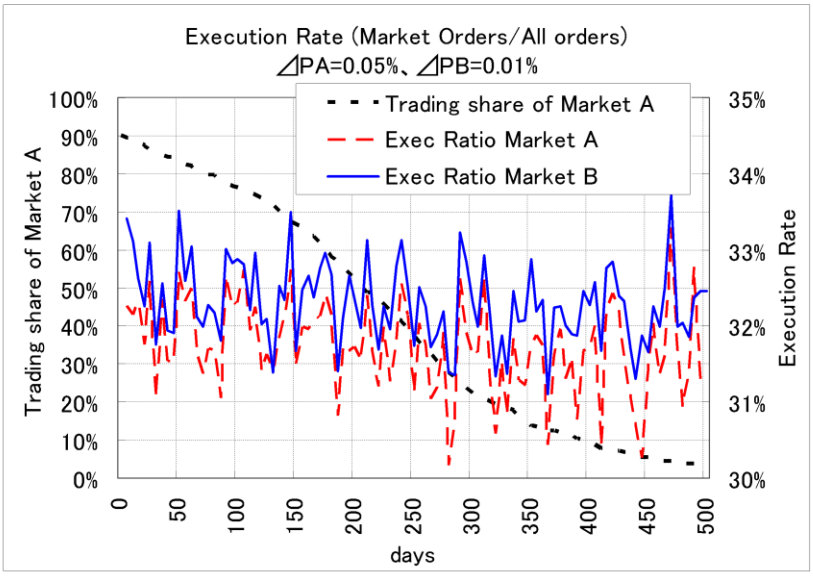
We modeled an Order Price, Po, by Random variables of Uniformly distributed in the interval from Expected Price, Pe, minus constant, Pd, to Pe plus Pd.

And then,

When Po lager than Pe, the agent orders to sell one unit.

When Po smaller than Pe, the agent orders to buy one unit.

Executions Rate (Market Orders/All Orders)



Execution Rate of Market B was slightly bigger than that of Market A. Because of the difference, Market B took the share