Thank you very much. I’m Takanobu MIZUTA from SPARX Asset Management. I’m also belonging to The University of Tokyo.

Today, I’m going to give a presentation under the title of This.
Financial exchanges sometimes employ a “price variation limit”, which restrict trades out of certain price ranges to avoid sudden large price fluctuations, “overshoots” overshots occur in bubble and crash in real financial market.

There is a debate over whether the price variation limit makes financial market more efficient or not.

To investigate price variation limits, we built an artificial market model implementing a learning process.

Experiment 1, we will show that the model should be implemented the learning process to replicate overshoot.

Experiment 2, We will also show that a parameters’ condition of the price variation limit to prevent overshoot.
First, I will describe our artificial market model.
Artificial Market Model (Agent Based Model)

Chiarella et. al. [2009]

- **Continuous Double Auction**
  - to implement realistic price variation limit
- **Agent model is Simple**
  - to avoid arbitrary result “Keep it short and simple”

heterogeneous 1000 agents

\[ r_{e,j}^t = \frac{1}{\sum_i w_{i,j}} \left( w_{1,j} \log \frac{P_j}{P^t} + w_{2,j} r_{h,j}^t + w_{3,j} \varepsilon_j^t \right) \]

Learning Process **Our Original**

- ↑ need to replicate an overshoot
  - Good Performance Strategy \( w_{i,j} \) is up
  - Bad Performance Strategy \( w_{i,j} \) is down

We built an artificial market model on basis of Chiarella et. al. [2009].

- Pricing mechanism is Continuous Double Auction
  - It is not simple, but, we need to implement realistic price variation limit
- Agent Model is Simple. This is to avoid arbitrary result, “Keep it short and simple” principle.

We think Artificial Market Models should explain Stylized Facts as Simply as possible.

There are heterogeneous 1000 agents. All agents calculate Expected Return using this equation.

And, the strategy weights are different for each agent

- First term is a Fundamental Strategy: When the market price is smaller than the fundamental price, an agent expects a positive return, and vice versa.
- Second term is a technical strategy: When historical return is positive, an agent expects a positive return, and vice versa.
- Third term is noise.

Chiarella’s model did not include Learning Process, however.

We built Learning Process of agents, this is our Original.

We showed that learning process need to replicate an overshoot.

Agents are comparing Historical Return and Each Strategy’s Return.

- When the strategy’s return and Historical Return are Same Sign, Good Performance Strategy, The strategy’s Weight is Up.
- When the strategy’s return and Historical Return are Opposite Sign, Bad Performance Strategy, The strategy’s Weight is Down.
Next, I show simulation results of Experiment 1 about learning process and replicating overshoot.
We examined two Cases, Case 1, Fundamental Value is constant, Case 2, Fundamental value is rapid incremented like this. This is bubble inducing trigger.

For Each cases, we examined With learning process And Without learning process.
Therefore, we examined four cases in all.
Case 1: Fundamental Value = constant

This Figure shows time evolution of market prices in case 1, Fundamental Value is constant.

In both cases, With learning process and without learning process, the results are very similar.
The prices were small fluctuating around Fundamental Value, here, Ten Thousand
Case 2: Fundamental Value → rapid increment (bubble trigger)

This Figure shows time evolution of prices in case 2.
Fundamental value was changed at this time, increased to New Fundamental Value, Fifteen Thousand.
This is the bubble inducing trigger.

Without Learning Process, Black line, Overshooting was not occurred.

On the other hand, with Learning Process, Red line,
the price went far beyond the new fundamental value.
Only with learning process, Overshoot occurred.
### Traditional Stylized Facts

<table>
<thead>
<tr>
<th>kurtosis</th>
<th>case 1</th>
<th>case 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>non-learning</td>
<td>learning</td>
</tr>
<tr>
<td>lag 1</td>
<td>3.018</td>
<td>5.394</td>
</tr>
<tr>
<td>lag 2</td>
<td>0.134</td>
<td>0.125</td>
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<tr>
<td>lag 3</td>
<td>0.101</td>
<td>0.105</td>
</tr>
<tr>
<td>lag 4</td>
<td>0.076</td>
<td>0.087</td>
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<tr>
<td>autocorrelation coefficient for square return</td>
<td>0.060</td>
<td>0.074</td>
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<tr>
<td>5</td>
<td>0.052</td>
<td>0.061</td>
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<td>0.048</td>
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<tr>
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<td>0.030</td>
<td>0.045</td>
</tr>
<tr>
<td>9</td>
<td>0.026</td>
<td>0.039</td>
</tr>
</tbody>
</table>

All cases replicated: Fat Tail and Volatility Clustering

This Table lists Traditional stylized facts in each case. In all cases, both kurtosis and autocorrelation for square returns for all i are positive. This means that all cases replicate Traditional stylized facts: fat-tail and volatility-clustering.
Hazard Rate (similar to “run test”)  
New Stylized fact to verify model replicating overshoot  

\[ H(i) \] conditional probability that sequence of positive return ends at \( i \), given that it lasts until \( i \).  

For Example \( i=3 \), \( H(3) \)  

<table>
<thead>
<tr>
<th>Time</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Return</td>
<td>▼</td>
<td>▼</td>
<td>▼</td>
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</tr>
</tbody>
</table>

\[ H(3) \]: Probability of 4th Return Negative  

Empirical Studies:  
Any cases: \( H(i) < 50\% \),  
Overshoot period: \( H(i) \) decline with \( i \) rapidly  
McQueen and Thorley [1994], Chan et. al. [1998]  
⇒ Overshoot returns tend to continue to be positive  
this tendency stronger continuing positive returns longer  

We propose Hazard Rate as New Stylized fact to verify model replicating overshoot  
Hazard Rate \( H_i \) is conditional probability that sequence of positive return ends at \( i \), given that it lasts until \( i \).  

For Example \( i=3 \), \( H_3 \) means like this.  
1\textsuperscript{st}, positive return, 2\textsuperscript{nd}, positive, 3\textsuperscript{rd} positive,  
In this condition, \( H_3 \) is probability of 4\textsuperscript{th} return become negative.  

Empirical Studies showed that, Any cases, \( H_i \) for most of \( i \) are smaller than 50\%  
And when including overshoot period, \( H_i \) decline rapidly with \( i \),  
This show that the overshoot returns tend to continue to be positive  
And this tendency stronger continuing positive returns longer
New Stylized Facts: Hazard Rate H(i)

<table>
<thead>
<tr>
<th></th>
<th>case 1</th>
<th></th>
<th>case 2</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>non-learning</td>
<td>learning</td>
<td>non-learning</td>
<td>learning</td>
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<td>55%</td>
<td>56%</td>
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<td>2</td>
<td>55%</td>
<td>52%</td>
<td>55%</td>
<td>50%</td>
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<td>3</td>
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<td>50%</td>
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<tr>
<td>4</td>
<td>54%</td>
<td>49%</td>
<td>52%</td>
<td>40%</td>
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<tr>
<td>hazard rate</td>
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<td>45%</td>
<td>48%</td>
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<td></td>
<td>6</td>
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<td>44%</td>
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<td>7</td>
<td>52%</td>
<td>41%</td>
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<tr>
<td></td>
<td>9</td>
<td>53%</td>
<td>40%</td>
<td>30%</td>
</tr>
</tbody>
</table>

Only with Learning process → Verified by Hazard Rate
And Only Case 2 with Learning ⇒ Replicating Overshoot

This Table lists New Stylized Facts: Hazard Rate in each case.

In case 2 with learning, hazard rate declined rapidly.
This case can replicate a significant Overshoot like actual markets.
On the other hand, the case without learning, hazard rate dose not declined rapidly.
The case can not replicate Overshoot.

Case 1, without learning, Hazard rates are upper 50% for all i.
This is Not consistent with empirical study.
On the other hand, Case 1, with learning, Hazard rates for most of i are smaller than 50%, even when price fluctuations are stable.

This consistent with empirical study.

Therefore, only cases with Learning Process were verified by Hazard Rate, and only Case 2 can replicate overshoot.
## Result Summary Experiment 1: Learning and Overshoot

<table>
<thead>
<tr>
<th>Case1</th>
<th>Case2</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Fundamental Value</strong>&lt;br&gt; = constant</td>
<td><strong>Fundamental Value</strong>&lt;br&gt; → rapid increment</td>
</tr>
<tr>
<td><strong>Without Learning Process</strong>&lt;br&gt; Not-Consistent with Empirical study</td>
<td><strong>With Learning Process</strong>&lt;br&gt; Consistent with Empirical study</td>
</tr>
<tr>
<td><strong>Stable</strong>&lt;br&gt; Not-Verified by Hazard Rate</td>
<td><strong>Stable</strong>&lt;br&gt; Verified by Hazard Rate</td>
</tr>
<tr>
<td><strong>No-Overshoot</strong>&lt;br&gt; Not-Verified by Hazard Rate</td>
<td><strong>Overshoot</strong>&lt;br&gt; (Bubble &amp; Crush)&lt;br&gt; Verified by Hazard Rate</td>
</tr>
</tbody>
</table>

Result Summary Experiment 1 relationship between Learning process and replicating Overshoot

The cases With learning process, both case 1 and case 2, were Consistent with Empirical study verified by Hazard Rate.

And case 2 can replicate overshoot, bubble and crush

The cases Without Learning Process were Not consistent with Empirical study Not verified by Hazard Rate.
Next, I show Simulation Results about Price Variation Limit.
We modeled the price variation limit like this.

There are two constant parameters. Tpl is a limit time span, and \( |P_{pl}| \) is limit price range.

We referred market price before \( t_{pl} \), \( P_{t_{pl}} \),
and any agents can not order outside from \( P_{t_{pl}} - |P_{pl}| \) to \( P_{t_{pl}} + |P_{pl}| \).

Concretely, any buy order prices above here, they are changed to this price.
and any sell order prices under here they are changed to this price.
This Figure shows time evolution of prices in case 2 with learning, comparing the case implemented price variation limit and not implemented.

Overshoot was vanished in the case implemented price variation limit.
However, price took longer to reach new fundamental price.

In the case not implemented price variation limit, price took faster to reach new fundamental price.
Implemented case, slower to reach new fundamental value, converging is slower.
Overshooting Amplitude and Converging Speed

reach time to new fundamental value

max price from new fundamental value

Preventing Overshooting and Immediate Converging → no market achieves both at once
Optimization of $t_{pl}$ and $\Delta P_{pl}$

Next, I investigated relationship between Overshooting Amplitude and Converging Speed.
I measured, here, reach time to new fundamental value, and here, max price from new fundamental value.

We want Preventing Overshooting and Immediate Converging. However, No market achieves both at once.
Therefore, it is important to Optimize of these Two parameters.
Max Price from New Fundamental Value

<table>
<thead>
<tr>
<th>max price from new fundamental value</th>
<th>1000</th>
<th>2000</th>
<th>3000</th>
<th>4000</th>
<th>7000</th>
<th>10000</th>
<th>15000</th>
<th>20000</th>
<th>25000</th>
<th>30000</th>
<th>40000</th>
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<tbody>
<tr>
<td>t&lt;sub&gt;pl&lt;/sub&gt;</td>
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<td>3,311</td>
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<td>3,494</td>
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<td>3,229</td>
<td>3,578</td>
<td>2,633</td>
<td>1,388</td>
<td>681</td>
<td>421</td>
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<td>3,320</td>
<td>3,231</td>
<td>1,345</td>
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</tr>
</tbody>
</table>

Blue area: \( \Delta P_{pl} \) < \( v \) \( \approx 0.128 \)  
\( v \): Converging Speed without price variation limit

Preventing Overshooting

\( \Delta P_{pl} / t_{pl} \) smaller (to upper right) \( \Rightarrow \) Overshoot smaller

\[ \text{NEED to prevent overshoots} \quad \Delta P_{pl} / t_{pl} < v \]

This table lists Max Price from New Fundamental Value in various T<sub>pl</sub> and \( \Delta P_{pl} \)

Blue area, \( \Delta P_{pl} \) over T<sub>pl</sub> is smaller than V  
V is a converging speed without price variation limit, approximately 0.128.

As you see, in this area, max price small, this means preventing overshoot.

This is smaller, to upper right area, overshoot are smaller.  
Therefore, we found that this is smaller than V is needed to prevent overshoot.
### Reach time to New Fundamental Value

<table>
<thead>
<tr>
<th>reach time to new fundamental value (x 1000)</th>
<th>$t_{pl}$</th>
<th>1000</th>
<th>2000</th>
<th>3000</th>
<th>4000</th>
<th>7000</th>
<th>10000</th>
<th>15000</th>
<th>20000</th>
<th>25000</th>
<th>30000</th>
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<tbody>
<tr>
<td>100</td>
<td>55</td>
<td>104</td>
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<td>216</td>
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<td>47</td>
</tr>
</tbody>
</table>

#### Delta Pl

Blue area: $\frac{\Delta P_{pl}}{t_{pl}} < v \approx 0.128$

$v$: Converging Speed without price variation limit

$\Delta P_{pl} / t_{pl}$ smaller (to upper right) $\Rightarrow$ converging speed slower

Should Avoid Not to be converging slower $\Delta P_{pl} / t_{pl} <\ll v$

---

This table lists Reach time to New Fundamental Value in various $T_{pl}$ and $\Delta P_{pl}$

Blue area, this is smaller than $V$.

This is smaller, to upper right area, converging speed are slower.

Therefore, we found that it this is too smaller than $V$ is should be Avoid Not to be converging slower.
Result Summary Experiment 2: about Price Variation Limit

Price Variation Limit
prevent Overshooting and Converging Slower

Optimization of $t_{pl}$ and $\Delta P_{pl}$

$\Delta P_{pl} / t_{pl}$ smaller $\Rightarrow$ Overshooting smaller

NEED to prevent overshoots

$\Delta P_{pl} / t_{pl} < v$

$\Delta P_{pl} / t_{pl} << v$

Should Avoid Not to be converging slower

$\Delta P_{pl} / t_{pl}$ smaller $\Rightarrow$ converging speed slower

$v$: Converging Speed
without price variation limit

Result Summary Experiment 2 about Price Variation Limit.

Price Variation Limit prevents Overshoot and cause Converging speed Slower to new fundamental value
Therefore, it is important to Optimize of these two parameters.

When this is smaller, overshoooting smaller.
We found that it needs smaller than, V to prevent overshoots

On the other hand, this is smaller, converging speed is slower.
We found that it should be avoid too smaller than V Not to be converging slower
I summarize this presentation

- To investigate price variation limits, we built an artificial market model implementing a learning process.

- Experiment 1, we showed that the model should be implemented the learning process to replicate overshoot.

- Experiment 2, We also showed that a parameters’ condition of the price variation limit to prevent overshoot.

http://www.slideshare.net/mizutata/cifer2013
That’s all for my presentation.

Thank you very much for your cooperation!

http://www.slideshare.net/mizutata/cifer2013

Could you say that again? (もう一度、おっしゃっていただけますか？)
I don’t quite understand your question. (ご質問の趣旨が良く分からないのですが)
Could you please rephrase your question? (ご質問を分かりやすく言い換えていただけますか)
So, you are asking me about.... (つまり、お尋ねの内容は...ですね)
I totally agree with you. (私も全くあなたと同意見です)
That’s a very challenging question for me to answer. (それは私にとって非常に答えがいのある質問です)
That’s a question I’m not sure I can answer right now. (その他ご質問にすぐお答えできるかどうか分かりません)
It would require further research. (さらなる研究結果を待ちたい)
You are right on that point. (その点に関してはあなたが正しい)
Our method will not solve the problem. (我々の方法ではその問題は解決できない)
Appendix
Artificial Market Model (Agent Based Model)

On basis of Chiarella et. al. [2009]
+ Learning Process of agents
  comparing between the case with Learning Process and without

Feature of our model
○ agent model is Simple
  → to avoid arbitrary result “Keep it short and simple”
  Models should explain stylized facts as simply as possible

● pricing mechanism is Continuous Double Auction
  → not simple to implement realistic price variation limit

☆ Learning process
  → agents switch strategy, fundamental or technical
  An overshoot occurred in the case with the learning process
  but did not occur in the case without the process

We built an artificial market model on basis of Chiarella et. al. [2009].
Chiarellla’s model did not include Learning Process, however.
We built Learning Process of agents.
And we are comparing between the case with Learning Process and without it.

Our Agent Model is Simple. This is to avoid arbitrary result, “Keep it short and simple” principle.
We think Artificial Market Models should explain Stylized Facts as Simply as possible,

Our pricing mechanism is Continuous Double Auction
It is not simple, but, we need to implement realistic price variation limit

Learning process
Here, Learning process means agents are switching strategy, fundamental strategy or technical strategy.
We will show that, an Overshoot occurred in the case With the learning process, however, overshoot did not occur in the case WithOut the process
Next, I will describe agent model.

All agents calculate Expected Return using this equation.

First term is a Fundamental Strategy:
When the market price is smaller than the fundamental price, an agent expects a positive return, and vice versa.

Second term is a technical strategy:
When historical return is positive, an agent expects a positive return, and vice versa.

Third term is noise.
After the expected return has been determined, an expected price is determined like this.
And, agents order base on this Expected Price.
Learning Process

Expected Return

\[
 r^t_{e,j} = \frac{1}{\sum_i w_{i,j}} \left( w_{1,j} \log \frac{P_j}{P^t} + w_{2,j} r^t_{h,j} + w_{3,j} \varepsilon^t_j \right)
\]

Historical Return

\[
 r^t_i = \log \frac{P^t}{P^{t-t_i}}
\]

Compare each Strategy

Same Sign

good performer Strategy

\[
 w_{i,j} \leftarrow w_{i,j} + kr^t_i [0, (w_{i,\text{max}} - w_{i,j})]
\]

Weight Up

With 1% probability:

Reset

\[
 w_{i,j} \leftarrow [0, w_{i,\text{max}}]
\]

Opposite Sign

bad performer Strategy

\[
 w_{i,j} \leftarrow w_{i,j} - kr^t_i [0, w_{i,j}]
\]

Weight Down

We also developed a model implementing a learning process of agents.

Agents are comparing Historical Return and each Strategy’ term, Fundamental strategy term, and Technical strategy term.

When the strategy’s expected return and Historical Return are Same Sign,

This means good performer Strategy.

The strategy’s Weight is Up.

When the strategy’s expected return and Historical Return are Opposite Sign,

This means bad performer Strategy.

The strategy’s Weight is Down.

We also added random learning.

In this way, agents learn better parameters and switch to the investment strategy that estimates correctly.
Next, agents determine order price and, buy or sell.

To Stabilize simulation runs for the continuous double mechanism, Order Prices must be covered widely in Order Book.

We modeled an Order Price, Po, by Random variables of Uniformly distributed in the interval from Expected Price, Pe, minus constant, Pd, to Pe plus Pd.

And then,
When Po lager than Pe, the agent orders to sell one unit.
When Po smaller than Pe, the agent orders to buy one unit.
### Hazard Rate

<table>
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<tr>
<th>$i$</th>
<th>non Price Limit</th>
<th>Price Limit</th>
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<tr>
<td>1</td>
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<td>49%</td>
</tr>
<tr>
<td>4</td>
<td>40%</td>
<td>47%</td>
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<tr>
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<tr>
<td></td>
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<td>29%</td>
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<td></td>
<td>7</td>
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</tr>
<tr>
<td></td>
<td>8</td>
<td>22%</td>
</tr>
<tr>
<td></td>
<td>9</td>
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</table>

Hazard Rates increased
→ Result by preventing bubble

Hazard Rate

Hazard Rates increased in implemented price variation limit
This Result shows preventing bubble
During overshooting, switching fundamental to technical
Consistent with empirical studies
[Frankel and Froot, 1990], [Yamamoto and Hirata, 2012]

I also show Strategies weight, in the case 2 with learning
which case include overshoot.

During overshooting, agents are switching fundamental strategy to technical
strategy.
This is consistent with empirical studies, like these studies.
Agents Not switching to the technical strategy → prevented overshooting.

I also show Strategies weight, in the case 2 with learning, implemented price variation limit

Agents are not switching fundamental strategy to technical strategy. This causes preventing overshooting, bubble and crash.