Investigation of Relationship between Tick Size and Trading Volume of Markets using Artificial Market Simulations

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Mechanism of Moving Share

Artificial Market Model
(Multi-Agent Simulation)

Competition Factors

Tick Size

Difficult to Change

Data: 2012
TSE and PTS

Empirical Analysis

Compare

Condition Not to Move Share

\( \Delta P_B > \Delta P_A \) or \( \bar{\sigma} > \Delta P_A \)

\( \bar{\sigma} \) depend on \( \Delta P \)
What is Tick Size?

Here, we define Tick Size $\Delta P = \text{Minimum Increment} / \text{Price}$

Difference of 1% Return is Serious Problem for some Investors
⇒ They prefer Stock Market has Smaller Tick Size $\Delta P$
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depend on \( \Delta P \) Moving Share

\[ \Delta P_B > \Delta P_A \quad \text{or} \quad \sigma > \Delta P_A \]

Compare

Empirical Analysis

Data: 2012 TSE and PTS

\( \sigma_t \)
Chiarella et. al. [2009]

- Continuous Double Auction
- Agent model is Simple

**heterogeneous 1000 agents**

**Expected Return**

\[
 r_{e,j}^t = \frac{1}{\sum_{i=1}^3 w_{i,j}} \left( w_{1,j} \log \frac{P_f}{P_{t-1}} + w_{2,j} r_{h,j}^{t-1} + w_{3,j} \xi_j^t \right)
\]

- **Fundamental**
- **Technical**
- **noise**

**Replicate Micro Structures**

- Trade number, Cancel rate, 1 day Volatility, and so on.

**Simulation Time ⇔ Real Time**

**convertible**

We interested in how long do markets need get shares.
Terms of Fundamental Strategy, Technical Strategy

**Fundamental Strategy Term**
- Fundamental Price > Market Price ⇒ expects + return
- Fundamental Price < Market Price ⇒ expects - return

**Technical Strategy Term**
- Historical Return > 0 ⇒ expects + return
- Historical Return < 0 ⇒ expects - return
To Stabilize simulation for continuous double mechanism, Order Prices must be covered widely in Order Book.
Agent Model Parameters

- **j**: agent number (1000 agents) ordering in number order
- **t**: tick time

**Expected Return**

\[
 r_{e,j}^t = \frac{1}{\sum_{i=1}^{3} w_{i,j}} \left( w_{1,j} \log \frac{P_f}{P^{t-1}} + w_{2,j} r_{h,j}^{t-1} + w_{3,j} \epsilon_j^t \right)
\]

**Parameters for agents**

- \( w_{i,j} \) and \( \tau_j \)
- Random of Uniform Distribution
  - \( w_{i,j} \)
    - \( i=1,3: 0 \sim 1 \)
    - \( i=2: 0 \sim 10 \)
  - \( \tau_j \)
    - \( 0 \sim 10000 \)

**Fundamental**

- **Fundamental Price**
  - \( P_f \)
    - \( 10000 = \text{constant} \)

**Technical**

- **Historical Return**
  - \( r_{h,j}^t = \log(P^t / P^{t-\tau_j}) \)

- **Random Price Noise**
  - \( \epsilon_j^t \)
    - Random of Normal Distribution
    - Average=0
    - \( \sigma=3\% \)

**Expected Price**

\[
 P_{e,j}^t = P^{t-1} \exp(r_{e,j}^t)
\]
**Market Order**

- Market Order: Choose the market list best price
- Limit Order: Allocate on basis of Historical Trading Volume Share of each market

**Agents**

- Market A
  - Initial Trading Volume Share: 90%
  - Tick Size: Large

- Market B
  - Initial Trading Volume Share: 10%
  - Tick Size: Small
Market Selection Model (example)

<table>
<thead>
<tr>
<th>Market A</th>
<th>Market B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sell</td>
<td>Sell</td>
</tr>
<tr>
<td>84</td>
<td>1</td>
</tr>
<tr>
<td>176</td>
<td>99.2</td>
</tr>
<tr>
<td>99</td>
<td>99.0</td>
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<tr>
<td>98</td>
<td>98.8</td>
</tr>
<tr>
<td>Price</td>
<td>Price</td>
</tr>
<tr>
<td>101</td>
<td>99.1</td>
</tr>
<tr>
<td>100</td>
<td>99.0</td>
</tr>
<tr>
<td>204</td>
<td>3</td>
</tr>
<tr>
<td>Buy</td>
<td>Buy</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1. Buy ¥98: Allocate on basis of Historical Trading Volume Share of each market
2. Buy ¥99.1: Market B
   \uparrow can buy ¥99.1 at Market B, immediately
3. Buy ¥100: Market B
   \uparrow can buy ¥99.1 at Market B, best price

Market B will take Trading Volume share because of (2), (3)
Allocate on basis of Historical Trading Volume Share

\[ Wa : \text{Probability an agent choose Market A} \]

\[ Ta, Tb : \text{Trading Volume of Market A or B within last } t_{AB} \]

\[ t_{AB} = 5 \text{ days} \]

\[ Wa = \frac{Ta}{Ta + Tb} \]
Competition between Stock Markets

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Tick Size
Difficult to Change

Competition Factors
Verification

Condition Not to Move Share

\[ \Delta P_B > \Delta P_A \quad \text{or} \quad \bar{\sigma} > \Delta P_A \]

\[ \sigma_t \]

depend on \( \Delta P \rightarrow \) Moving Share

Compare

Empirical Analysis

Data: 2012 TSE and PTS
Stylized Facts

<table>
<thead>
<tr>
<th>tick size(%)</th>
<th>0.0001%</th>
<th>0.001%</th>
<th>0.01%</th>
<th>0.1%</th>
<th>1%</th>
</tr>
</thead>
<tbody>
<tr>
<td>trade rate</td>
<td>23.5%</td>
<td>23.5%</td>
<td>23.4%</td>
<td>23.1%</td>
<td>22.1%</td>
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<tr>
<td>cancel rate</td>
<td>26.2%</td>
<td>26.2%</td>
<td>26.3%</td>
<td>26.6%</td>
<td>27.6%</td>
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<tr>
<td>number of trades / 1 day</td>
<td>6,361</td>
<td>6,358</td>
<td>6,345</td>
<td>6,279</td>
<td>6,081</td>
</tr>
<tr>
<td>standard deviations for 1 tick</td>
<td>0.05%</td>
<td>0.05%</td>
<td>0.05%</td>
<td>0.06%</td>
<td>0.16%</td>
</tr>
<tr>
<td>standard deviations for 1 day (20000 ticks)</td>
<td>0.59%</td>
<td>0.56%</td>
<td>0.57%</td>
<td>0.57%</td>
<td>1.15%</td>
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<tr>
<td>kurtosis</td>
<td>1.50</td>
<td>1.48</td>
<td>1.45</td>
<td>1.10</td>
<td>1.81</td>
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</tbody>
</table>

Autocorrelation coefficient for square return:

<table>
<thead>
<tr>
<th>lag</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
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<tr>
<td></td>
<td>0.229</td>
<td>0.141</td>
<td>0.109</td>
<td>0.091</td>
<td>0.078</td>
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<tr>
<td></td>
<td>0.228</td>
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<td>0.025</td>
<td>0.013</td>
<td>0.008</td>
<td>0.006</td>
<td>0.004</td>
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</table>

Replicate Fat-Tail and Volatility-Clustering

\[ \bar{\sigma}_t = 0.05\% \]

Volatility at tick size small

+ Replicate Micro Structures (Original)

Trade rate, Cancel rate, 1 tick and 1 day volatility

Simulation Time ⇔ Real Time convertible

We interested in how long do markets need get shares.
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\[ \bar{\sigma}_t \text{ depend on } \Delta P \rightarrow \text{Moving Share} \]

Competition between Stock Markets

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Mechanism of Moving Share

Compare

\[ \Delta P_B > \Delta P_A \text{ or } \bar{\sigma} > \Delta P_A \]
Tick Size of Market B $\Delta PB=0.01\%$, Tick Size is not small

Tick Size of Market A, $\Delta PA$ is larger, Market A is taken trading volume share faster
Execution Rate of Market B was slightly bigger than that of Market A. Because of the difference, Market B took the share
Market B can hardly take the share in spite that \( \Delta PA \) is very larger than \( \Delta PB \)
### Tick Size Condition Not to Move Share

<table>
<thead>
<tr>
<th>Trading share of Market A at 500 days</th>
<th>$\Delta P_B$</th>
<th>0.0001%</th>
<th>0.0002%</th>
<th>0.0005%</th>
<th>0.001%</th>
<th>0.002%</th>
<th>0.005%</th>
<th>0.01%</th>
<th>0.02%</th>
<th>0.05%</th>
<th>0.1%</th>
<th>0.2%</th>
</tr>
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<tr>
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<td></td>
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<td>91%</td>
<td>91%</td>
<td>92%</td>
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<td>0.002%</td>
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<td>96%</td>
<td>100%</td>
</tr>
</tbody>
</table>

### Condition Not to Move Share

- $\Delta P_B > \Delta P_A$
- $\bar{\sigma} > \Delta P_A$

$\bar{\sigma}_t = 0.05\%$
Competition between Stock Markets

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\( \sigma \) depend on \( \Delta P \)

Moving Share

Mechanism of Moving Share

Compare

Empirical Analysis

Data: 2012 TSE and PTS
Relationship between $\sigma_t$ and Share ($\Delta PB$ is enough small)

When $\sigma_t$ depends on $\Delta PA$, Market A is taken share very Rapidly
\( \sigma_t < \Delta P_A \)

unable trading in Market A

\( \rightarrow \) many trading in Market B

\( \Rightarrow \) trading share moving to Market B

\( \sigma_t > \Delta P_A \)

needless Market B

\( \Rightarrow \) trading share not moving
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(Multi-Agent Simulation)

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Factors

Verification

Tick Size

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Condition Not to Move Share

\[ \Delta P_B > \Delta P_A \quad \text{or} \quad \frac{1}{10} > \Delta P_A \]

\[ \sigma_t \text{ depend on } \Delta P \to \text{Moving Share} \]

Compare

Empirical Analysis

Data: 2012 TSE and PTS

Mechanism of Moving Share
**Data**

**Data Period:** All business days in calendar year 2012  
**Universe:** 439 stocks  
  Selected by TOPIX 500 index whole data period  
  they had same tick size for every month ends  
  they were traded every business days at least once

**Horizontal Axis:** Tick Size of TSE $\Delta P$ for each stock  
  $\Delta$: standard deviation of 10 seconds return for each stock, $\sigma_t$  
  ●: trading volume share in PTS for each stock

**Summarize Markets:**

- **Traditional Stock Exchanges:**  
  Tokyo Stock Exchange, Osaka SE,  
  Nagoya, Fukuoka, Sapporo, and JASDAQ

- **PTS (Proprietary Trading System):**  
  Japan Next PTS J-Market, Japan Next PTS X-Market,  
  and Chi-X Japan PTS
Empirical Result

Right Side, Volatility $\sigma_t$ depends on Tick Size $\Delta P$, Tokyo Stock Exchange is taken share more.

(right vertical axis is reversed)
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Summary
Appendix
Definition of Market/Limit order

In this study

A little difference from actual market

All agents decide an order price

<table>
<thead>
<tr>
<th>sell</th>
<th>order book</th>
<th>buy</th>
</tr>
</thead>
<tbody>
<tr>
<td>sell</td>
<td>price</td>
<td>buy</td>
</tr>
<tr>
<td>84</td>
<td>101</td>
<td></td>
</tr>
<tr>
<td>176</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>99</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>98</td>
<td>77</td>
<td></td>
</tr>
</tbody>
</table>

Exist matching order
Order executed immediately

No matching order
Order not executed immediately

Agents decide an order price,
if exist matching order, market order else limit order