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Abstract

We investigated competition, in terms of taking market share of trading volume, between two artificial financial markets that have exactly the same specifications except tick size, i.e., the minimum units of a price change per market price, and initial trading volume using multi-agent simulations. We found that market share of the trading volume of a market that adopts a tick size larger than $\overline{\sigma_t}$ (standard deviation of tick by tick return when tick size is small enough) is taken by another market that adopts a tick size smaller, and market share of the trading volume of a market that adopts a tick size smaller than approximately $\overline{\sigma_t}$ is rarely taken by another market. We compared these simulation results with empirical data of the Tokyo Stock Exchange. We argue that these investigations will encourage discussion about adequate tick sizes that markets should adopt.

* It should be noted that the opinions contained herein are solely those of the authors and do not necessarily reflect those of SPARX Asset Management Co., Ltd. and JPX group. Contact: Takanobu Mizuta (mizutata@gmail.com)

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1 Introduction

Recently, the number of stock markets that make full use of IT and achieve low-cost operations is increasing, especially in the United States and Europe. Their market shares of trading volume have caught up with those of traditional stock exchanges. Therefore, each stock is traded at many stock markets at once. Whether such fragmentation makes markets more efficient has been debated, for example, Foucault and Menkveld (2008); O'Hara and Ye (2011). There are many factors that cause the taking of market share of trading volume between actual markets, such as the minimum unit of a price change per market price(tick size), speed of trading systems, length of trading hours, stability of trading systems, safety of clearing, and variety of order types. It is said that the smallness of tick size is one of the most important factors to compete with other markets.

Because many factors result in the taking market share of trading volume of one actual market by another, an empirical study cannot isolate the relationship between tick size and market share of trading volume. Therefore, it is difficult to discuss the effect of tick size from only the results of empirical studies. An artificial market, which is a type of agent-based simulation, will help us determine effects on institutions and regulations in financial markets. Many studies have investigated the effect on several institutions and regulations using artificial market simulations Westerhoff (2008); Yeh and Yang (2010); Kobayashi and Hashimoto (2011); Thurner et al. (2012); Yagi et al. (2012); Mizuta et al. (2013). No simulation studies, however, were focused on investigating the effect of the relationship between tick size and market share of trading volume on stock markets using an artificial market model.

In this study, we investigated competition, in terms of taking market share of trading volume, between two artificial financial markets that have exactly the same specifications except of tick size and initial trading volume using an artificial market model. We also compared simulation results with empirical data obtained from the Tokyo Stock Exchange.

2 Artificial Market Model

We built a simple artificial market model on the basis of the models developed by Chiarella et al. (2009); Mizuta et al. (2013). The model treats only one risk asset and one non-risk asset (cash) and adopts a continuous double auction[*1] to determine the market price of a risk asset. The number of agents is n. First, at time $t = 1$, agent 1 orders to buy or sell the risk asset; then at $t = 2$ agent 2 orders to buy or sell. At $t = 3, 4, , , n$, agents $3, 4, , , n$ respectively order to buy or sell. At

[*1] A continuous double auction is an auction mechanism where multiple buyers and sellers compete to buy and sell financial assets in the market and where transactions can occur at any time whenever an offer to buy and an offer to sell match(Friedman (1993); Tokyo Stock Exchange (2012a)).

$t = n + 1$, going back to the first agent, agent 1 orders to buy or sell, and at $t = n + 2, n + 3, , n + n$, agents $2, 3, , , , n$ respectively order to buy or sell, and this cycle is repeated. Note that t passes even if no deals have occurred. An agent j determines an order price and buys or sells by the following process. Agents use a combination of the fundamental value and technical rules to form expectations on a risk asset return. The expected return of agent j is

$$r^t_{e,j} = \frac{1}{\sum_{i=1}^{3} w_{i,j}} \left(w_{1,j} \log \frac{P_f}{P^{t-1}} + w_{2,j} r^{t-1}_{h,j} + w_{3,j} \epsilon^t_j \right), \tag{1}$$

where $w_{i,j}$ is the weight of term i of agent j, which is independently determined by random variables of uniform distribution inside the interval $(0, w_{i,max})$ at the start of the simulation for each agent, P_f is a fundamental value that is constant, P^t is the market price of the risk asset at time t. (When dealing does not occur at t, P^t remains at the last market price P^{t-1}, and at $t = 1$, $P^t = P_f$), ϵ^t_j is noise determined by random variables of normal distribution with an average 0 and variance σ_ϵ, $r^t_{h,j}$ is a historical price return inside an agent's time interval τ_j, and $r^t_{h,j} = \log{(P^t / P^{t-\tau_j})}$ in which τ_j is independently determined by random variables of uniform distribution inside the interval $(1, \tau_{max})$ at the start of the simulation for each agent. The first term of Equation (1) represents a fundamental strategy: an agent expects a positive return when the market price is lower than the fundamental value and vice verse. The second term of Equation (1) represents a technical strategy: an agent expects a positive return when the historical market return is positive and vice verse. After the expected return has been determined, the expected price is

$$P^t_{e,j} = P^{t-1} \exp{(r^t_{e,j})}. \tag{2}$$

We modeled an order price $P^t_{o,j}$ by random variables of normal distribution in an average $P^t_{e,j}$, a standard deviation P_σ, where P_σ is a constant. A minimum unit of a price change is ΔP, and we round off a fraction of less than ΔP. To buy or sell is determined by the magnitude of the relation between the expected price $P^t_{e,j}$ and the order price $P^t_{o,j}$, that is,

When $P^t_{e,j} > P^t_{o,j}$, the agent orders to buy one share,

When $P^t_{e,j} < P^t_{o,j}$, the agent orders to sell one share.

Agents always order only one share. Our model adopts a continuous double auction, so when an agent orders to buy (sell), if there is a lower price sell order (a higher price buy order) than the agent's order, dealing immediately occurs. We call such an order "market order". If there is not a lower price sell order (a higher price buy order) than the agent's order, the agent's order remains in the order book. We call such an order "limit order". The remaining order is canceled after t_c from the order time. Agents can short sell freely. The quantity of holding positions is not limited, so agents can take any shares for both long and short positions to infinity.

We investigated the situation in which agents can trade one kind of stock in two stock markets. The two stock markets have exactly the same specifications except minimum unit of a price change, $\Delta P_A, \Delta P_B$ and initial share of trading volume, W_A, W_B. The agents should decide to which market they order, market A or market B.

Our market choice model mentioned following chooses a market as same way as a smart order routing (SOR) which is a kind of algorithm trading used in real financial markets Credit Swiss AES Japan (2013); Goldman Sachs India (2013), and also same as the model of Adhami (2010).

When the agent order is buy (sell), the agent searches the lowest sell (highest buy) orders of each market. We call these prices "best prices". When best prices differ between two markets and the order will be a market order in least one of the markets, the agent orders to buy (sell) in a market in which the best price is better, *i.e.,* lower (higher) in the case of the buy (sell) order. In other cases, *i.e.,* when the best prices are exactly the same or the order will be a limit order in both markets, the agent orders to buy (sell) in market A with probability $W_A = T_A/(T_A + T_B)$, where T_A is the trading volume of market A within last t_{AB}, calculating span of W_A and T_B is that of market B. Therefore, $W_B = 1 - W_A = T_B/(T_A + T_B)$ is the probability the agent orders to buy (sell) in market B. Before time reaches t_{AB}, W_A remains constant at an initial value. To summarize, if the market order and best prices differ, agents order to buy (sell) in the market in which the best price is better than in another market. In other cases, agents order to buy (sell) in markets depending on the market share of trading volume.

3 Simulation Results

We searched for adequate model parameters verified by statistically existing stylized facts and market micro structures to investigate the effect of tick size difference to competition between stock markets. We found parameters to replicate both long-term statistical characteristics and very short-term micro structures of real financial markets. Specifically, we set, $n = 1000, w_{1,max} = 1, w_{2,max} = 10, w_{3,max} = 1, \tau_{max} = 10000, \sigma_\epsilon = 0.06, P_\sigma = 30, t_c = 20000, t_{AB} = 100000$ and $P_f = 10000$. We ran simulations to $t = 10000000$.

3.1 Verification of Model

Table 1 lists statistics for various tick sizes in which there is only one market. All statistics are averages of 100 simulation runs. Tick size is the ratio of the minimum unit of a price change ΔP to the fundamental value P_f. We define 20000 time steps as 1 day because the number of trades within 20000 time steps is almost the same as that in actual markets per day. Trades rates and cancel rates [*2] are similar in actual markets and verify the model. Standard deviations for one day

[*2] We referred to the statistics of actual markets in Tokyo Stock Exchange (2011) and Bloomberg. Trade rate is the ratio of the number of trades to that of all orders + cancels. Cancel rate is the ratio of the number of cancels to that of all

Table1 Stylized facts for various tick sizes

	tick size(%)	0.0001%	0.001%	0.01%	0.1%	1%
about trading	trade rate	23.5%	23.5%	23.4%	23.1%	22.1%
	cancel rate	26.2%	26.2%	26.3%	26.6%	27.6%
	number of trades / 1 day	6,361	6,358	6,345	6,279	6,081
standard deviations	for 1 tick	0.05%	0.05%	0.05%	0.06%	0.16%
	for 1 day (20000 ticks)	0.59%	0.56%	0.57%	0.57%	1.15%
kurtosis		1.50	1.48	1.45	1.10	1.81
autocorrelation coefficient for square return	lag					
	1	0.229	0.228	0.228	0.210	0.025
	2	0.141	0.141	0.141	0.120	0.013
	3	0.109	0.108	0.108	0.090	0.008
	4	0.091	0.091	0.091	0.075	0.006
	5	0.078	0.078	0.078	0.064	0.004

are also similar to those in actual markets and also verify the model. We also found that standard deviations increased with a tick size of 1%.

In many previous artificial market studies, the models were verified in terms of whether they can be used to explain stylized facts such as fat-tail and volatility-clustering[*3]. Table 1 also lists the stylized facts in each case. We used returns for about 10 seconds (100 time unit intervals) to calculate the statistical values for the stylized facts[*4]. In all runs, we found that both kurtosis and autocorrelation coefficients for square returns with several lags were positive, which means that all runs replicate stylized facts. However, only when the tick size was 1%, the autocorrelation coefficients for square returns with several lags were almost zero. We believe the reason is that the standard deviation of returns is almost constant because the tick size is too big and most returns for about 10 seconds will be zero or 1%.

This indicates that our model can replicate very short term micro structure, trades rates, cancel rates and standard deviations of returns for one tick and replicate long-term statistical characteristics observed in real financial markets. Therefore, the model is verified to investigate the effect of tick size difference to competition between stock markets.

orders + cancels

*3 Excellent reviews are LeBaron (2006); Chen et al. (2012). Fat-tail means that the kurtosis of price returns is positive. There have been empirical studies on fat-tail, Mandelbrot (1963); Cont (2001). Volatility-clustering means that the square returns have positive autocorrelation, and the autocorrelation slowly decays as its time separation becomes larger. There have been empirical studies on volatility-clustering, Cont (2001); Sewell (2006).

*4 In this model, time passes by an agent just ordering even if no dealing occurred. Therefore, the returns for one tick (one time) include many zero returns, which will bias statistical values. This is the reason we use returns for about 10 seconds.

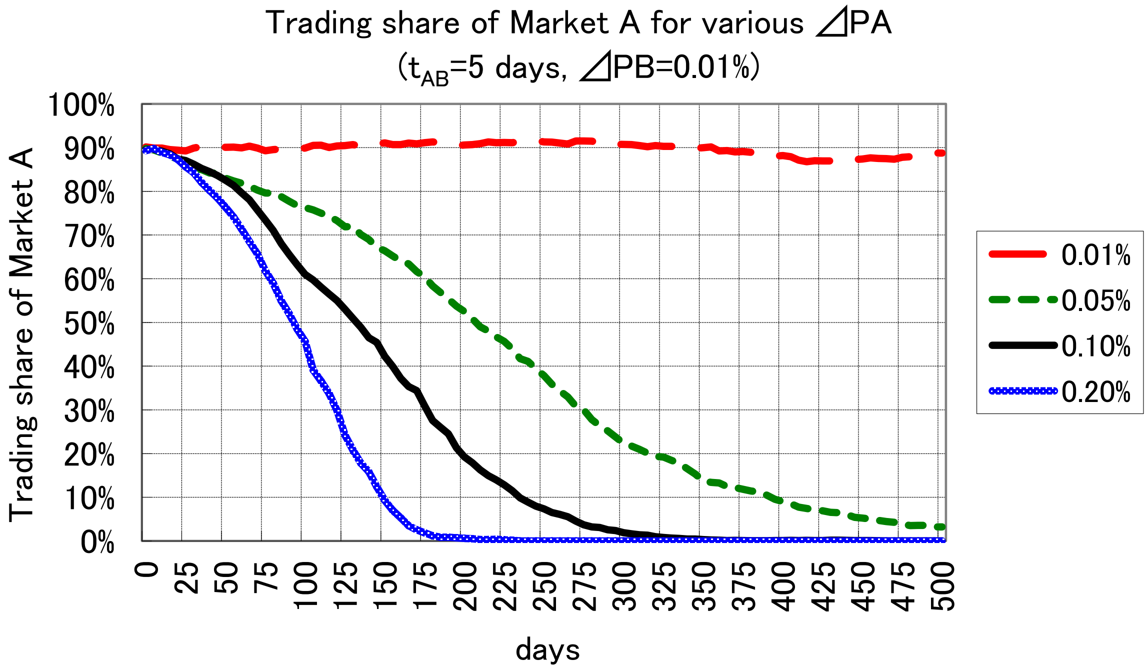

Figure1 Time evolution of market share of trading volume in Market A for various $\Delta P_A(\Delta P_B = 0.01\%)$

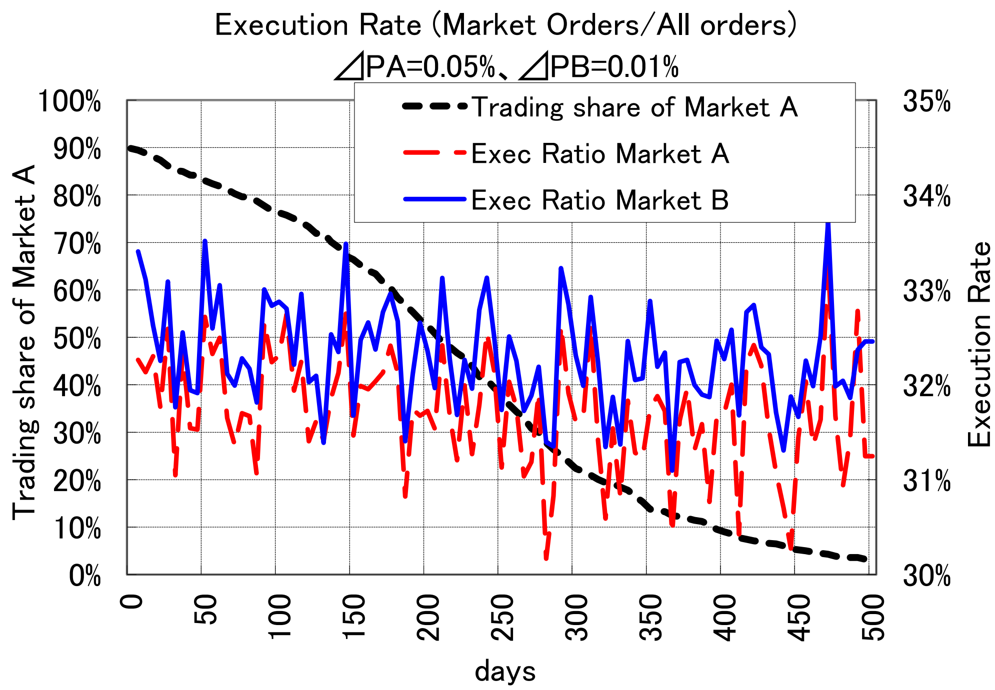

Figure2 Time evolution of execution rates (trade rates)

3.2 Time Evolutions of Market Share of Trading Volume

We investigated the transition of market shares of trading volume involving two markets. The two stock markets, markets A and B, had exactly the same specifications except minimum unit

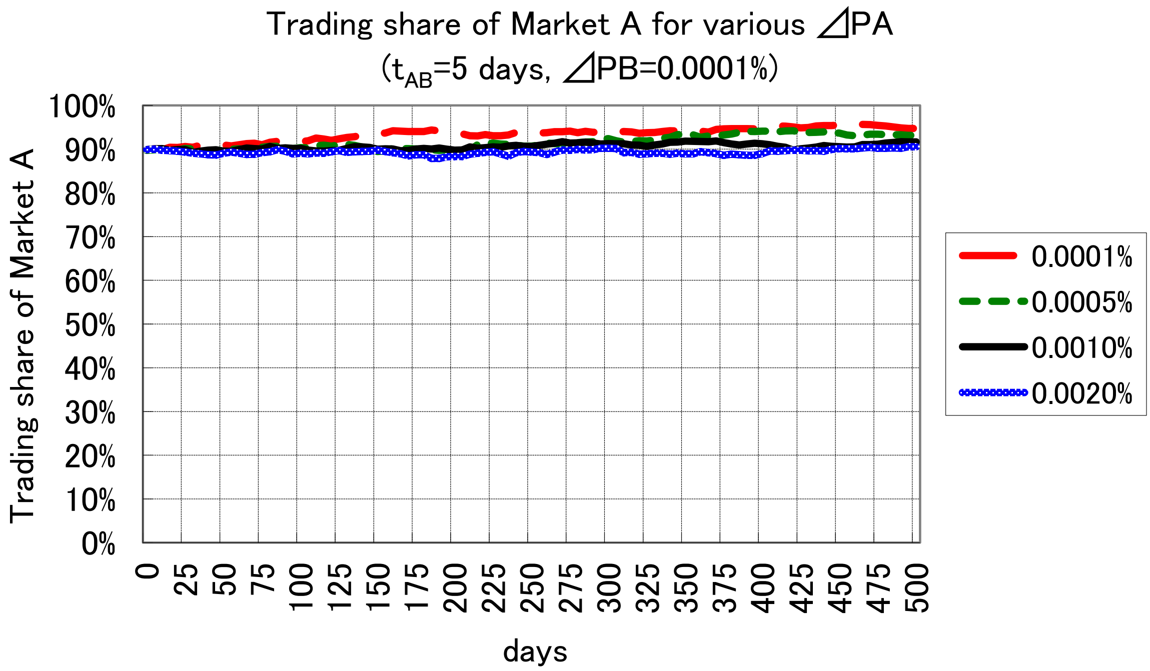

Figure3 Time evolution of market share of trading volume in Market A for various $\Delta P_A(\Delta P_B = 0.0001\%)$

of a price change, $\Delta P_A, \Delta P_B$ and initial market share of trading volume, W_A, W_B. Figure 1 shows the time evolution of market share of trading volume of market A for various ΔP_A. We set initial $W_A = 0.9$ and $\Delta P_B = 0.01\%$. We found that when ΔP_A was larger, market B took market share of trading volume from market A faster.

Figure 2 shows the time evolution of execution rates (trade rates) of market A and B respectively in the case of $\Delta P_A = 0.05\%$ and $\Delta P_B = 0.01\%$. The execution rates of market B were always higher than those of market A. This means that the prices of waiting limit orders in market B were frequently better than those in market A because ΔP_B was smaller than ΔP_A. The difference of execution rates cased that market B took market share of trading volume from market A.

Figure 3 shows the case in which $\Delta P_B = 0.0001\%$, which is 1/100 compared with that in Figure 1, and ΔP_A also became 1/100. We found that market B could not take market share in spite of the fact that the ratios of ΔP_A to ΔP_B were the same as those in Figure 1. Therefore, competition under too small of tick sizes does not affect the taking of market share of trading volume.

3.3 Relationship between Tick Size and Taking Market Share

Next, we investigated the relationship between tick size and taking market share of trading volume. Table 2 lists the market share of trading volume of market A at 500 days, W_A for various ΔP_A and ΔP_B. Market share of trading volume was averaged over 100 simulation runs. Shading

Table2 Market share of trading volume of Market A at 500 days for various ΔP_A and ΔP_B

Trading share of Market A at 500 days		⊿PB										
		0.0001%	0.0002%	0.0005%	0.001%	0.002%	0.005%	0.01%	0.02%	0.05%	0.1%	0.2%
⊿PA	0.0001%	90%	90%	91%	91%	92%	94%	97%	99%	100%	100%	100%
	0.0002%	90%	90%	90%	91%	91%	94%	97%	99%	100%	100%	100%
	0.0005%	89%	90%	91%	91%	92%	94%	96%	99%	100%	100%	100%
	0.001%	89%	89%	90%	90%	92%	94%	97%	99%	100%	100%	100%
	0.002%	87%	88%	89%	89%	91%	93%	97%	99%	100%	100%	100%
	0.005%	84%	85%	85%	84%	87%	92%	96%	99%	100%	100%	100%
	0.01%	75%	76%	76%	77%	78%	83%	92%	98%	100%	100%	100%
	0.02%	53%	52%	53%	54%	54%	59%	70%	93%	100%	100%	100%
	0.05%	5%	5%	4%	5%	5%	5%	6%	23%	93%	100%	100%
	0.1%	0%	0%	0%	0%	0%	0%	0%	0%	0%	94%	100%
	0.2%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	96%

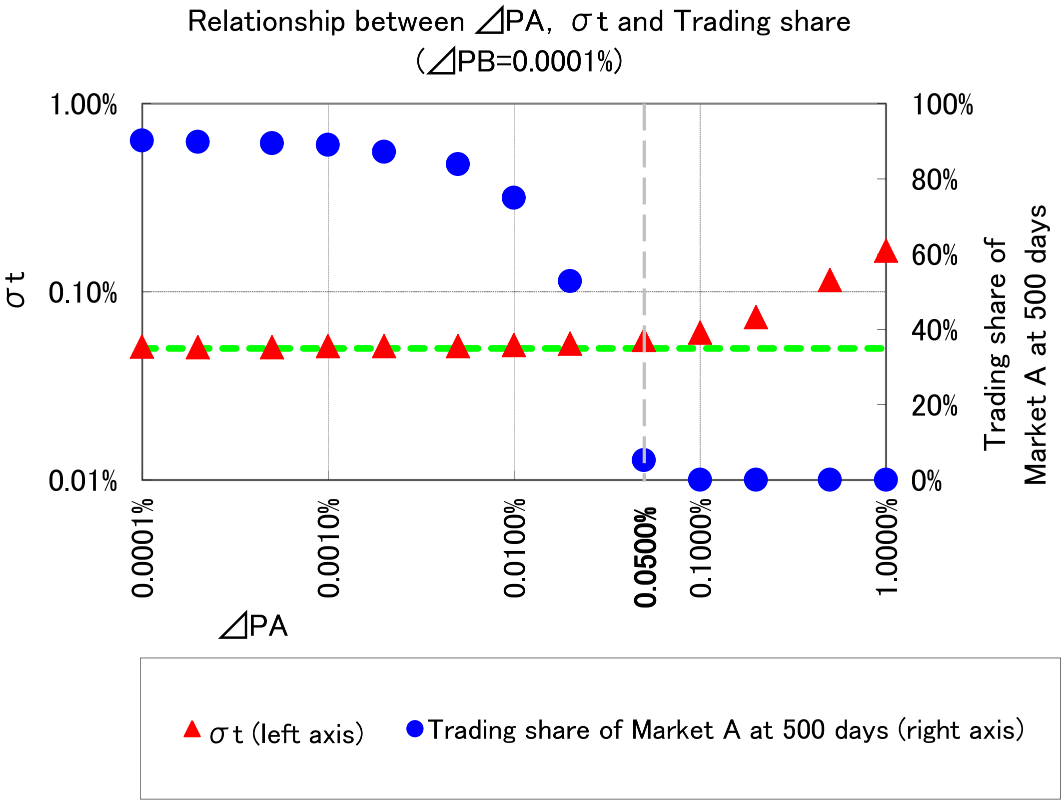

Figure4 Standard deviation of returns for 1 tick, σ_t and market share of trading volume of Market A at 500 days for various ΔP_A

denotes $W_A < 10\%$. We also drew the following two border lines,

$$\Delta P_A \leq \Delta P_B \text{ (dashed line)}, \tag{3}$$
$$\Delta P_A < \overline{\sigma_t} \simeq 0.05\% \text{ (solid line)}, \tag{4}$$

where $\overline{\sigma_t}$ is the standard deviation of return for one tick, which was found approximately 0.05% from Table 1. Region Equation (3) satisfies the region above the dashed line in Table 2, and region Equations (4) satisfy the region above the solid lines. The region that at least Equation (3) or (4)

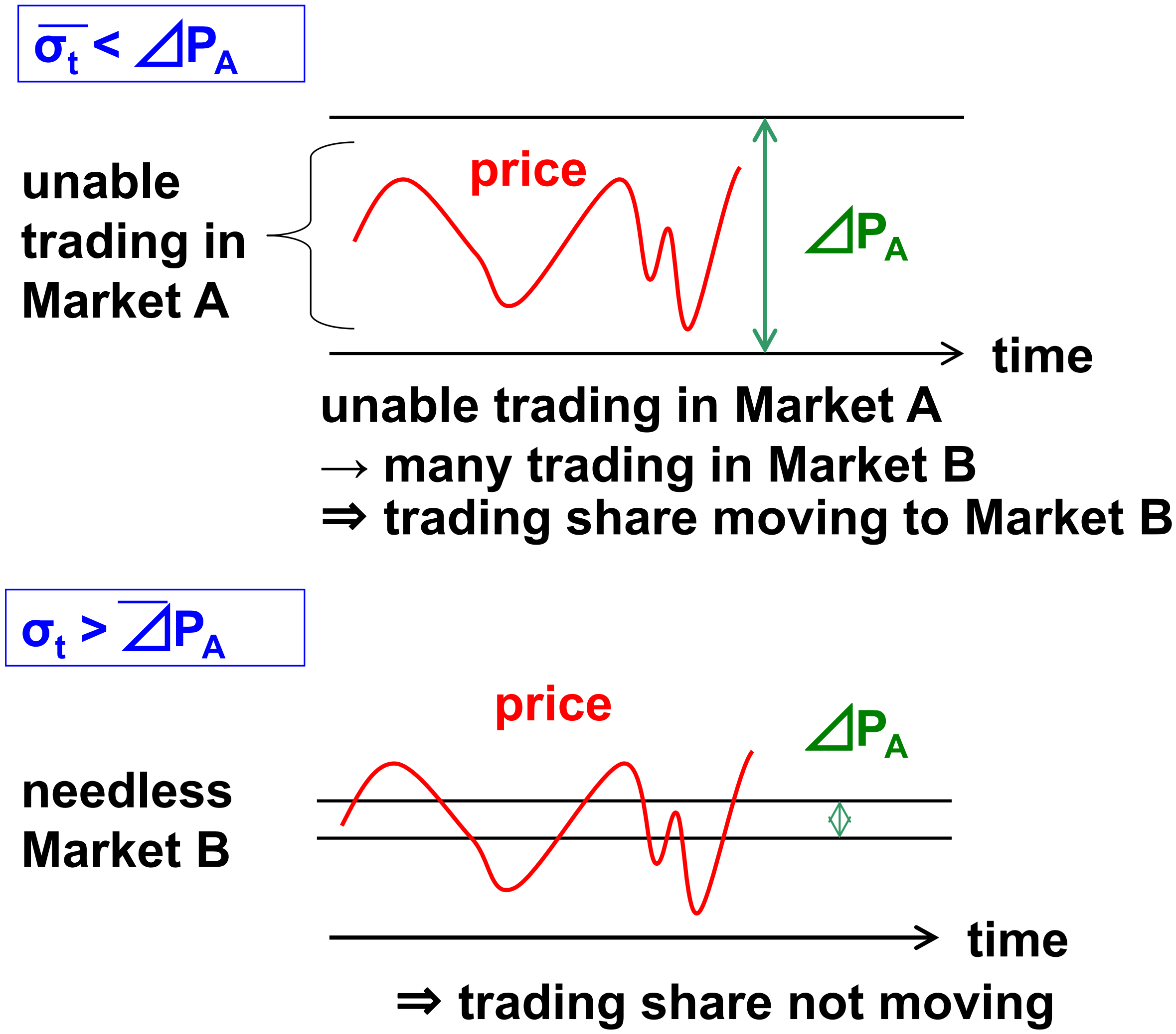

Figure5　Summary of relationship between tick sizes and standard deviations of returns for 1 tick

satisfies was not shading, and market share of trading volume of market A was rarely taken. The region that both Equations (3) and (4) did not satisfy, under the dashed and double lines, was mostly darkened, and market share of trading volume of market A was rapidly taken. In the region above the solid line, Equation (4), market share of trading volume of market A was rarely taken even if ΔP_B was much smaller than ΔP_A and did not depend on ΔP_B. This means that when the tick size of market A is smaller than $\overline{\sigma_t}$, market share of trading volume of market A is rarely taken.

Figure 4 shows the standard deviation of return for one tick, σ_t, and market share of trading volume of market A at 500 days, W_A for various ΔP_A where ΔP_B = 0.0001%. σ_t and W_A are averages of 100 simulation runs. The left vertical axis and horizontal axis are logarithmic scales. Equation (3) was never satisfied for all ΔP_A in Figure 4. The horizontal dotted line is $\sigma_t = \overline{\sigma_t}$, and the vertical dashed line is $\Delta P_A = \overline{\sigma_t}$. On left side of $\Delta P_A = \overline{\sigma_t}$, σ_t did not depend on ΔP_A. This means that the difference in tick size did not affect price formations where tick sizes were smaller than approximately $\overline{\sigma_t}$. On the other hand, on the right side of $\Delta P_A = \overline{\sigma_t}$, ΔP_A was larger, σ_t was larger. This implies that the prices normally fluctuated less than ΔP_A; however, price variation less than ΔP_A was not permitted. Therefore, price fluctuations depended on ΔP_A. In this case, market share of trading volume of market A rapidly deceased according to increasing ΔP_A. On the left side of $\Delta P_A = \overline{\sigma_t}$, however, market A was not taken the share rapidly.

We summarize discussion so far referring Figure 5. When ΔP_A is larger than approximately $\overline{\sigma_t}$ (Figure 5 top), if ΔP_B is smaller than ΔP_A, there is a large amount of trading in market B inside ΔP_A. Therefore, market B takes market share of trading volume from market A. On the other hand, when ΔP_A is smaller than approximately $\overline{\sigma_t}$ (Figure 5 bottom), even if ΔP_B is very small, price fluctuations cross many widths of ΔP_A and sufficient price formations are occur only in market A. Therefore, market B can rarely take market share of trading volume from market A.

4 Empirical Analysis

Next, we analyzed empirical data and compared them with the simulation results shown in Figure 4, using Japanese stock market data.

The data period included all business days in the 2012 calendar year. A number of stocks analyzed is 439, which were selected by TOPIX 500 [*5] over the entire index data period, had the same minimum unit of a price change for every month end and were traded every business day. Figure 6 shows the standard deviations for 10 seconds of each stock, where σ_t (triangles) is the averaged standard deviation of return for 10 minutes except opening prices for every day and ΔP is market share of the trading volume of the Proprietary Trading System (PTS) [*6] of each stock (circles) with tick sizes that are minimum unit of a price change divided by averaged prices at the end of every month of the Tokyo Stock Exchange. We used the data from the Tokyo stock exchange to calculate ΔP and σ_t. We used Bloomberg data to calculate the market share or trading volume of the PTS, which is its entire trading volume divided by those of Japanese traditional stock exchanges and PTS, where PTSs are Japan next PTS J-Market, X-Market, and Chi-X Japan PTS, and where Japanese traditional stock exchanges are the Tokyo, Osaka, Nagoya, Fukuoka, and Sapporo stock exchanges and JASDAQ. The right vertical axis is upside down to easily compare with Figure 4. The horizontal dotted line is $\sigma_t = \overline{\sigma_t}$, and vertical dashed line is $\Delta P = \overline{\sigma_t}$.

Figure 6 indicates that ΔP was larger, σ_t was larger especially $\Delta P > 0.5\%$. These results are similar to those in Figure 4. The market share of trading volume of PTS deceased along with ΔP for the entire ΔP. This result is similar $\Delta P_A > \overline{\sigma_t}$ in Figure 4. We found that when ΔP was larger, PTS more easily took market share of trading volume, and σ_t tended to increase along with ΔP.

[*5] TOPIX 500 is a free-float capitalization-weighted index that is calculated based on the 500 most liquid and highly market capitalized domestic common stocks listed on the Tokyo stock exchange first section (Tokyo Stock Exchange (2012b)).

[*6] Electric trading systems outside stock exchanges are called PTSs in Japan. A PTS is very similar to an Alternative Trading System (ATS) and Electronic Communications Network (ECN) in other countries.

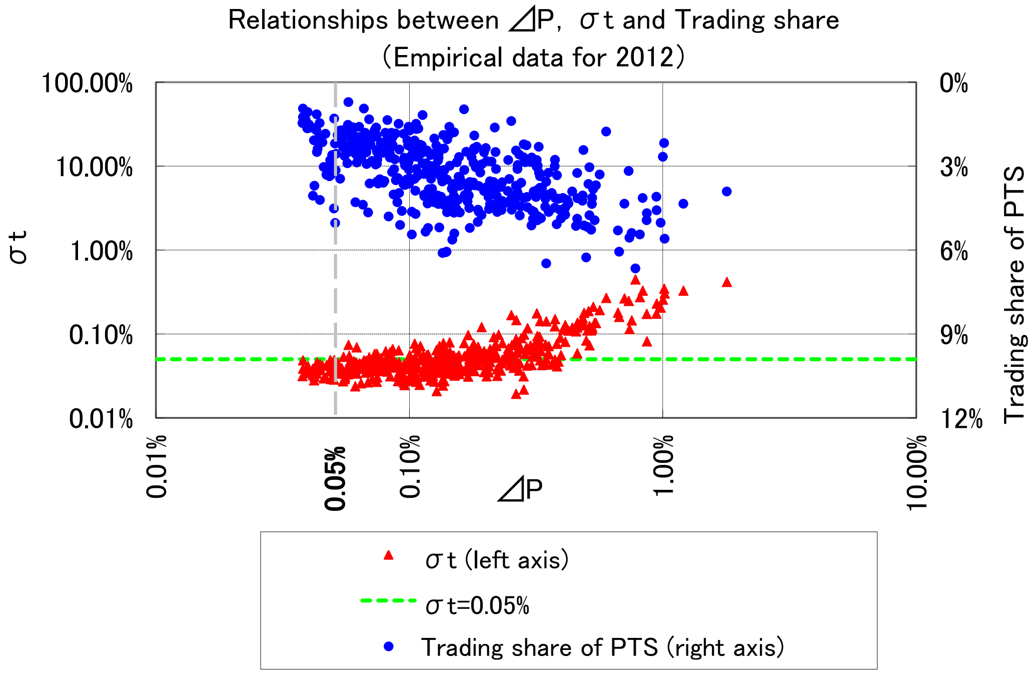

Figure6 Empirical analysis of relationship between tick sizes, standard deviations of returns, and market share of trading volume of PTS

5 Conclusion

We investigated competition, in terms of taking market share of trading volume between two artificial financial markets that had exactly the same specifications except tick size and initial trading volume using multi-agents simulations. When the tick size of market A, ΔP_A, was larger than approximately the standard deviation of tick by tick return when tick size was small enough, $\overline{\sigma_t}$ (Figure 5 top), if the tick size of market B, ΔP_B, was smaller than ΔP_A, much trading occurred in market B inside ΔP_A. Therefore, market B took market share of the trading volume from market A. On the other hand, when ΔP_A was smaller than approximately $\overline{\sigma_t}$ (Figure 5 bottom), even if ΔP_B was very small, price fluctuations cross many widths of ΔP_A and enough price formations occurred only in market A. Therefore, market B could rarely take market share of trading volume from market A. We also compared these simulation results with empirical data from the Tokyo Stock Exchange. We argued that this investigation will enable discussion about the adequate tick sizes markets should adopt.

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