Why do Active Funds that Trade Infrequently Make a Market more Efficient? 
– Investigation using Agent-Based Model

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Abstract—Since managers of active funds choose stocks that are expected to raise their prices on the basis of the fundamental value, many argue that active funds discover the fundamental value and make a market more efficient. However, it has not been clear whether actual active funds make a market more efficient or not. It has been shown that active funds that trade infrequently earn more. At first glance, infrequent trades seem to not impact and change market prices and this leads to market prices not converging with the fundamental price. Therefore, it is important to discuss whether active funds that trade infrequently make a market more efficient or not, and if so, we should investigate the mechanism of how they do so. In this study, we built a model of investors who trade infrequently in an artificial market model, and we investigated effects of these investors on market prices and whether they make a market more efficient by using the model. The results indicate that such active investors trade frequently in the rare situation that the market becomes unstable and inefficient due to the market price moving away from the fundamental price. These trades, occurring only at a necessary time, impact the market prices and lead them converging with the fundamental price. This leads preventing the market from becoming more unstable and less efficient. Though the trading volume of fundamental investors is low throughout whole period, the volume increases greatly only when a market becomes less efficient, and these trades then make the market efficient. An increasing market volatility makes the order prices of speculators (technical investors) move further away from the fundamental price, and this leads to amplifying market volatility more excessively. It is possible that the orders of active investors prevent this amplification. This also implies that money moving from active funds to passive funds leads a market to become less efficient.

I. INTRODUCTION

There are two types of portfolio management strategies for funds that invest in stocks and/or bonds, “active” managed funds, where a manager chooses stocks expected to raise their prices and invests in them, and “passive” managed funds, where a manager expects a return the same as a market index, e.g., the Dow Jones industrial average and S&P 500, and replicates the components of the index. Recently, the assets of passive funds have increased; however, those of active funds have decreased because, as some empirical studies [1], [2] have argued, the average return of active funds lost that of passive funds, and recent regulations have made fund sellers accountable for fund costs, especially in the United States [3].

Since active funds invest in stocks on the basis of the intrinsic value of companies (fundamental value), there are many arguments made that they discover the fundamental value and this leads to market prices converging with the fundamental price (make a market more efficient); therefore, they play an important role in allocating capital, which is an important function in capitalism [4]. Furthermore, some claim that the rise in passive funds threatens to fundamentally undermine the entire system of capitalism and market mechanisms that facilitate an increase in the general welfare [5].

However, it has not been clear whether actual active funds make a market more efficient or not and how much the assets of active funds are needed to make a market more efficient. In addition, an empirical study [6] showed that active funds that trade infrequently earn more. This implies that the assets of these funds will increase, while those of active funds that frequently trade will decrease. At first glance, infrequent trades seem to not impact and change market prices and thus not lead to market prices converging with the fundamental price. Therefore, it is important to discuss whether active funds that trade infrequently make a market more efficient or not, and if so, we should investigate the mechanism of how they do so.

In fact, there are opposing arguments. On the one side, since active funds that perform well measure fundamental value more precisely, these funds make a market more efficient even though they trade infrequently [7], [8], and on the other side, funds that trade infrequently do not make a market more efficient [9]. Anyways, the mechanism by which funds that trade infrequently make a market more efficient is poorly understood, although several mechanisms have been suggested. An empirical study [10] showed that the volume of active funds varies over time and that funds earn when the volume is larger, which implies the mechanism of how funds

1Since passive funds need not research companies, costs of passive fund are cheaper than those of active funds [1], [2].
that trade infrequently make a market more efficient.

Such discussion on the mechanism between the micro-macro feedback of certain types of investors is very difficult only using the results of empirical studies. Empirical studies cannot be conducted to investigate situations that have never occurred in actual financial markets, such as ones in which passive investors are more than present. Furthermore, because so many factors cause price formation in actual markets, an empirical study cannot be conducted to isolate the direct effect of changing the distribution of investor types on price formation.

An artificial market, which is a kind of agent-based model, can handle situations that have never occurred, such as ones in which passive investors are more than present, and can isolate the direct effect of changing the distribution of investor types on price formation. These are strong advantages for an artificial market simulation. The effects of the distribution and of several changing regulations have been investigated by using artificial market simulations [11]–[14].

Not only academies but also financial regulators and stock exchanges have recently become interested in agent-based models such as artificial market models to investigate financial markets. Indeed, an article in Science by Battiston et al. [15] stated that "since the 2008 crisis, there has been increasing interest in using ideas from complexity theory (using network models, agent-based models, and so on) to make sense of economic and financial markets.", and an article in Nature by Farmer and Foley [16] stated that "such (agent-based) economic models should be able to provide an alternative tool to give insight into how government policies could affect the broad characteristics of economic performance, by quantitatively exploring how the economy is likely to react under different scenarios"

Many studies have investigated the effects of some kinds of investors on price formation and the effects of several changing regulations and rules by using artificial market simulations, for example, leveraged ETFs [17], high-frequency traders (HFTs) [18]–[22], arbitrage traders between markets that have different latencies [23], market impacts [24], [25], financial market crashes [26]–[28], price variation limits [29]–[31], frequent batch auctions [32], dark pools [33]–[35], investor networks and herding [36], the increasing speed of order matching systems on financial exchanges [37], market efficiency [38], and the rules for investment diversification [39].

Indeed, the effects of market makers and passive funds were investigated by using artificial market simulations [40]. However, the reason active investors who trade infrequently affect market prices and make a market more efficient has not been investigated. In the first place, investors who trade infrequently have not been modeled in artificial market models.

Therefore, in this study, we built a model of agents who trade infrequently in the artificial market model and investigated the effects of active investors who trade infrequently on market prices and whether they make a market more efficient by using the model.

Fig. 1. Time evolution of market prices \( P_t \) when number of fundamental agents \( N_F = 0, 100, 500 \).

Fig. 2. Distribution of frequencies of market price ranges and trading volume of fundamental agents for various market price ranges when number of fundamental agents \( N_F = 500 \).

II. ARTIFICIAL MARKET MODEL

We modeled agents who reflected the characteristics of investors who trade infrequently. The simplicity of the model is very important for this study because unnecessarily replicating macro phenomena leads to models that are over-fitted and too complex, and such models would prevent us from understanding and discovering the mechanisms that affect price formation because the number of related factors would increase. We explain the basic concept for constructing our artificial market model in “Basic Concept for Constructing Model” in the Appendix.

A. Agent Model

The number of all agents is \( N \). First, half of all agents \( N/2 \) have one share of stock, and the other half have cash
of agents who trade infrequently. In the following sections, we explain the rules for determining an order price for each type of agent.

1) Fundamental Agents: The number of fundamental agents is \( N_F \). The order price of an agent \( j \) at time \( t \), \( P_{o,t,j} \) is

\[
P_{o,t,j} = P_f \exp(d \sigma^j \pm m \mu^j),
\]

where \( d \) and \( m \) are constant, and \( \sigma^j \) is determined by random variables that follow a standard normal distribution for each agent \( j \). \( \mu^j \) is determined by random variables that are uniformly distributed in \((0, 1)\) for each agent \( j \). The \( \pm \) gives \(-\) in the case of a buy order and \(+\) in the case of a sell order.

The fundamental agents determine order prices \( P_{o,t,j} \) by depending not on the latest market price \( P_{t-1} \) but on the fundamental price \( P_f \), which is the intrinsic value of a company. The agents do not know \( P_f \) but try to estimate it. \( d \sigma^j \) is a ratio of the difference between \( P_f \) and the estimated fundamental price to \( P_f \). Fundamental investors generally want to buy (sell) at a sufficiently lower (higher) price than their estimated fundamental prices, and such a sufficient difference of prices is called the “margin of safety” [41]. \( m \mu^j \) is the ratio of the margin of safety to the estimated fundamental price.

2) Technical Agents: The number of technical agents is \( N_T \). A half of the technical agents \( N_T/2 \) adopts a momentum strategy, and the other half adopts a contrarian strategy.

The order price of a momentum strategy agent \( j \) at time \( t \), \( P_{o,t,j} \) is

\[
P_{o,t,j} = P_t \frac{P_t}{P_{t-1+m\tau}}.
\]

The technical agents decide order price same as an expected price, this leads an expected return is \( \ln(P_{o,t,j}/P_t) \). In the momentum strategy, they expect that the expected return, \( \ln(P_{o,t,j}/P_t) \) equals the historical return, \( \ln(P_t/P_{t-1+m\tau}) \). This leads the equation (2).
That of a contrarian strategy agent is
\[ P_{o,j}^t = P_t - tm^j, \]
where \( tm^j \) is a natural number determined by random variables uniformly distributed in \((1, tm_{\text{max}})\) for each agent \( j \), and \( tm_{\text{max}} \) is a constant. In the contrarian strategy, they expect that the expected return, \( \ln(P_{o,j}^t/P^t) = \text{const.} \), equals the inverses of historical return, \( -\ln(P^t/P_{o,j}^t) \). This leads to the equation (3).

Previous studies showed that such technical agents are needed to replicate price formations observed in real financial markets [12](see also “Verification of Model” in Appendix.) This is the reason we introduced the technical agents.

3) Noise Agents: The number of noise agents is \( N_N \). The order price of an agent \( j \) at time \( t \), \( P_{o,j}^t \) is
\[ P_{o,j}^t = P^t \exp(\eta \sigma^{t,j}), \]
where \( \eta \) is constant, \( \sigma^{t,j} \) is determined by random variables that follow a standard normal distribution for each time \( t \) and agent \( j \). In this study, we handle a stock traded at a high enough volume. We introduce noise agents to supply enough liquidity. If there are no noise agents, order prices sometimes incline to one side heavily, and this leads to orders not being matched. Also, in real financial markets, there are such many liquidity suppliers [42].

4) Note on Modeling Passive Investors: In this study, passive agents are not introduced. Passive funds, where a manager expects a return the same as a market index and replicates the components of the index, trade only when they are subscribed to or redeemed, or when names included in a tracking index are changed. In this study, we do not deal with these trades, so passive agents never trade in this model. This means that this study cannot handle cases in which only the number of passive investors increases. This means that a decrease in the number of fundamental agents \( N_F \) corresponds to money moving from active funds to passive funds because passive agents never trade. Therefore, we can interpret this study as an investigation into price formation in the case of money moving from active funds to passive funds.

B. Price Determination Model

After all agents determine orders, buy or sell and an order price, the market price \( P^t \) is determined by a “call auction” [43] where the numbers of sell/buy orders at prices lower/higher than \( P^t \) are matched. In a call market, buy and sell orders are grouped together and then executed at specific times rather than executed one by one continuously. The market price is determined at the crossing point of supply and demand curves. The supply(demand) curve is a cumulative number of orders that sellers(buyers) want to sell over(buy under) a price.

III. Simulation Results

Specifically, we set \( N_T = 100, N_N = 1000, C_0 = P_f = 10000, d = 0.1, m = 0.1, tm_{\text{max}} = 100 \), and \( \eta = 0.1 \). We ran simulations until \( t = t_e = 10000 \). We explain the verification of these parameters in “Verification of Model” in Appendix. The model has only the necessary minimum parameters and hardly obtains arbitrary results.

We compared several statistical values of the simulation runs for \( N_F = 0, 10, 20, 50, 100, 200, 300, 400, \) and 500 not only under other parameters that were fixed but also the same random number table. As mentioned in section II-A-4, a decrease in the number of fundamental agents \( N_F \) corresponds to money moving from active funds to passive funds because passive agents never trade. Therefore, a decrease in \( N_F \) means that the number of fundamental agents operating as active funds decreases and that of passive agents operating as passive funds increases.

We introduce the parameter “market inefficiency” \( M_{ie} \) for directly measuring market efficiency.
\[ M_{ie} = \frac{1}{t_e} \sum_{t=1}^{t_e} \frac{|P^t - P_f|}{P_f}. \]
where \( || \) is an absolute value. \( M_{ie} \) is always greater than zero, and \( M_{ie} = 0 \) means a market is perfectly efficient\(^2\). The larger the \( M_{ie} \), the less efficient the market\(^3\).

Figure 1 shows the time evolution of market prices \( P^t \) when the number of fundamental agents was \( N_F = 0, 100, \) and 500. The higher the number of fundamental agents, the more efficient the market became because \( P^t \) oscillated in a narrower range near the fundamental price \( P_f = 10000 \). Table I lists the market inefficiency \( M_{ie} \) for various \( N_F \) and shows that the higher the \( N_F \) and lower the \( M_{ie} \), the more efficient the market became. This indicates the possibility that a decrease in the number of active investors makes a market less efficient, and this implies that money moving from active funds to passive funds leads to a market becoming less efficient. As we mentioned in section II-A-4, note that this study cannot deal with the case in which only the number of passive investors increases.

Table II lists the trading volume share (total volume of a type of agent/total volume of all types of agents) for various \( N_F \). The trading volume of fundamental agents was much smaller than those of other type agents.

To discuss why fundamental agents whose trading volume is low make a market more efficient, we show two figures. Figure 2 shows a distribution of the frequencies of market price ranges and the trading volume of fundamental agents for various market price ranges, and Fig. 3 show a distribution of the frequencies of the average of the absolute returns and the trading volume of fundamental agents for various market price ranges; both figures are for the case of \( N_F = 500 \).

\(^2\)Even though we calculated market inefficiency, we did not intend to discuss an efficient market hypothesis. In our model, the market is not efficient because of the existence of technical agents.

\(^3\)This index is sometimes used in experimental financial studies of people, in which this market inefficiency is sometimes called “relative absolute deviation” (RAD) [44]. Many indications for measuring market efficiency have been proposed [45]. A feature of \( M_{ie} \) is that it is calculated directly by using a fundamental price \( P_f \), which is never observed in empirical studies. We can also use \( M_{ie} \) in simulation and experimental studies because we can exactly define \( P_f \).
Figure 2 indicates that the fundamental agents traded very near \( P^t = 9800 \), far from \( P_f \) (solid line), though the frequency at \( P^t = 9800 \) was low (dashed line). This means that the fundamental agents traded frequently when the market became inefficient as \( P^t \) moved farther away from \( P_f \).

Figure 3 indicates that the absolute return near \( P^t = 9800 \) was large (dashed line) and that market volatility (dispersion of returns) increased. In short, the fundamental agents traded frequently when market prices sharply declined and market volatility was excessive. This trading behavior is consistent with a previous empirical study [10]. However, this mechanism of making a market more efficient is inconsistent with the mechanisms argued by [7], [8].

These results indicate that the fundamental agents traded frequently in the rare situation that a market becomes unstable and inefficient due to the market price getting away from the fundamental price. These trades, occurring only at a necessary time, impact the market prices and lead them converging with the fundamental price. This leads to preventing the market from becoming more unstable and less efficient. Though the trading volume of fundamental agents was low throughout the whole period, the volume increased a lot only when the market became less efficient, and these trades then made the market efficient. It is implied that money moving from active funds to passive funds leads a market to become less efficient because these trades of active funds decrease.

From equation (2), an increasing market volatility makes the order prices of momentum strategy agents move further away from \( P_f \), and this leads to amplifying market volatility more excessively. It is possible that the orders of fundamental agents prevent this amplification. Future work should investigate the details of this mechanism.

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The averages of the profits for fundamental agents were higher than those of the other types of agents. The higher the number of fundamental agents \( N_F \), the less the fundamental agents earn less, but this amount is higher than zero. This suggests that there are more opportunities for fundamental agents to earn when the market becomes less efficient.

IV. CONCLUSION AND FUTURE WORK

In this study, we built a model of agents who trade infrequently in an artificial market model, and we investigated the effects of active investors who trade infrequently on market prices and whether they make a market more efficient by using the model.

Simulation results indicated that the fundamental agents traded frequently in the rare situation that the market becomes unstable and inefficient due to the market price moving further away from the fundamental price. These trades, occurring only at a necessary time, impacted the market prices and lead them converging with the fundamental price. This lead to preventing the market from becoming more unstable and less efficient. Though the trading volume of fundamental agents was low throughout the whole period, the volume increased a lot only when the market became less efficient, and these trades then made the market efficient. This trading behavior is consistent with a previous empirical study [10]. However, this mechanism of making a market more efficient is inconsistent with the mechanisms argued by [7], [8].

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The case in which only the number of passive investors increases is also important future work. This study cannot deal with this case since passive agents never traded in this model. However, actual passive funds trade when a fund is subscribed or redeemed, or when names included in a tracking index are changed. Therefore, investigating the effects of these trades is important.

For more detailed discussions, we should compare the simulation results to those from studies that use other methods, e.g., empirical studies and theoretical studies. An artificial market can deal with situations that have never occurred, such as passive investors being more than present, and can isolate the direct effect of changing the distribution of investor types on price formation. These are strong advantages for an artificial market simulation; however, the outputs of these simulations may not be accurate or credible forecasts for actual markets. It is an important role for artificial market simulations to reveal the possible mechanisms that affect price formation through many runs to gain new insights; conversely, one limitation of artificial market simulations is that their outputs may, but not certainly, occur in actual financial markets.
APPENDIX

Basic Concept for Constructing Model

An artificial market, which is a kind of agent-based model, can be used to discuss investor distributions that have never been realized, can handle regulation changes that have never been made, and can isolate the pure contribution of these changes to price formation [11]–[14]. These are the strong points of the artificial market simulation.

However, the outputs of this simulation would not be accurate or credible forecasts of the actual future. The simulation needs to reveal possible mechanisms that affect price formation through many simulation runs, e.g., searching for parameters or purely comparing the before/after of changes. The possible mechanisms revealed by these runs will give us new intelligence and insight into the effects of the changes on price formation in actual financial markets. Other methods of study, e.g., empirical studies, would not reveal such possible mechanisms.

Indeed, artificial markets should replicate macro phenomena existing generally for any asset and any time. Price variation, which is a kind of macro phenomena, is not explicitly modeled in artificial markets. Only micro processes, agents (general investors), and price determination mechanisms (financial exchanges) are explicitly modeled. Macro phenomena emerge as the outcome interactions of micro processes. Therefore, the simulation outputs should replicate macro phenomena existing generally due to prove that simulation models are probable in actual markets.

However, it is not a primary purpose for an artificial market to replicate specific macro phenomena only for a specific asset or a specific period. An unnecessary replication of macro phenomena leads to models that are over-fitted and too complex. Such models would prevent us from understanding and discovering mechanisms that affect price formation because the number of related factors would increase.

Indeed, artificial market models that are too complex are often criticized because they are very difficult to evaluate [12]. A model that is too complex not only would prevent us from understanding mechanisms but also could output arbitrary results by over-fitting too many parameters. It is more difficult for simpler models to obtain arbitrary results, and these models are easier to evaluate.

Therefore, we constructed an artificial market model that is as simple as possible and do not intentionally implement agents to cover all the investors who would exist in actual financial markets.

Verification of Model

In many previous artificial market studies, the models were verified to see whether they could explain stylized facts, such as a fat tail or volatility clustering [11]–[14]. A fat tail means that the kurtosis of price returns is positive. Volatility clustering means that the square returns have positive autocorrelation, and this autocorrelation slowly decays as its lag becomes longer. Many empirical studies, e.g., that of Sewell [46], have shown that both stylized facts (fat tail and volatility clustering) exist statistically in almost all financial markets. Conversely, they also have shown that only the fat tail and volatility clustering are stably observed for any asset and in any period because financial markets are generally unstable.

Indeed, the kurtosis of price returns and the autocorrelation of the square returns are stably and significantly positive, but the magnitudes of these values are unstable and very different depending on the asset and/or period. The kurtosis of price returns and the autocorrelation of the square returns were observed to have very broad magnitudes of about $1 \sim 100$ and about $0.01 \sim 0.2$, respectively [46].

For the above reasons, an artificial market model should replicate these values as significantly positive and within a reasonable range as we mentioned. It is not essential for the models to replicate specific values of stylized facts because the values of these facts are unstable in actual financial markets.

Table IV lists the statistics, standard deviation of returns, kurtosis of price returns, and the autocorrelation coefficient for square returns when $N_F = 0$. This shows that this model replicated the statistical characteristics, fat tails, and volatility clustering, observed in real financial markets.

Disclaimer

Note that the opinions contained herein are solely those of the authors and do not necessarily reflect those of SPARX Asset Management Co., Ltd. and Nomura Research Institute, Ltd.

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